

Reduced Order Modelling for the Optimization of CSP Tower Receivers and their Cavities for High Temperature Applications

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Abstract. We present a Reduced Order Method approach to the heat exchange and losses in a simulated 3D cavity of CSP tower receivers. We validate the method in a 2D Boussinesq model problem for natural convection monitoring temperature, pressure and velocity for different values of the Rayleigh number. For the 3D problem of heat losses estimation we compute the snapshots with Ansys Fluent in a realistic model of a cavity with wind velocity and wall temperatures as varying parameters. The reduction in computational time can be up to four orders of magnitude with relative errors of 10^{-5} .

Keywords: Reduced Order Model, Tower-Cavity Heat Losses Optimization

1. Introduction

The modeling of heat transfer in solar plants depends on many parameters and geometrical variables: materials, dimensions, intensity and shape of the incident solar spot, carrier fluid pressure, mass flow... Additionally, there are many design constraints (performance, geometry, integrity of the structure) that must be fulfilled. Therefore, the optimization of the process is difficult and costly, and an iterative procedure is required to swap over the full parameter and variables range.

Reduced Order Modelling (ROM) provides reductions of several orders of magnitude in the computational cost required by the numerical simulation of parametric processes and design problems involving large numbers of degrees of freedom, making affordable the analysis of the behavior of highly complex systems, which would be out of reach using resolution techniques or standard numerical approximation (cf. [1] for a general overview).

Typically, ROMs are pre-built in an offline phase, requiring the full-order model to be solved for several well-chosen cases. Then, using appropriate techniques, this information (the “snapshots”) is compressed to select the basic or main component functions of the ROM, which are those used in the online phase.

ROM techniques are used in this work to address the problem of the design of CSP tower receivers and their cavities in the case of high temperature applications ($>600^{\circ}\text{C}$). Two coupled field thermal-fluid problems are to be solved in parallel, namely:

- **P1**: Heat transfer to HTF running through the irradiated pipes
- **P2**: Convective Heat Losses to ambient (with or without cavity)

The next step will be a nonlinear constrained optimization algorithm that uses the ROMs as core loop calculation both for the **P1** and **P2** problems and different cavities' geometries [2].

2. Reduced Order Modelling

ROMs are built in two phases, the offline phase and the online phase. In the offline phase, we need to solve the FOM in several well-chosen cases and then through different mathematical techniques we obtain our ROM basis in order to use it in the online phase. The online phase uses this basis to solve the reduced problem to obtain new predictions.

For P1, we plan to afford the reduced modelling of the parametric heat flow within the receiver panels by a hybrid (intrusive/non-intrusive) reduced-order approach. Actually, there exist two versions of ROMs method, the intrusive method and the data-driven or non-intrusive method. In the case of intrusive ones, the reduced solutions are determined by solving a reduced order model, i.e., a projection of the FOM (Full Order Model) onto the reduced space. However, the data-driven or non-intrusive reduced order models are independent from the original physical system. They only learn from the snapshots, i.e., either numerical approximations or measurements of the states of the dynamical systems, when the operator of the discretized systems are not available. In this work, we decide to combine both strategies, as explained hereafter in detail. We consider as parameters both the geometrical parameters determining the receiver geometry (diameter and length of the pipes) and the physical parameters determining the boundary data for the heat flow (incoming mass flow and radiation). To approach the problem, we started with a preliminary test based on the 2D Boussinesq equations for natural convection problem [3]. The reduced turbulence model starts from finite element snapshots for velocity, pressure and temperature computed by a Boussinesq VMS-LES Smagorinsky turbulence model [4] with LPS (Local Projection Stabilization) in pressure [5] (called the "full order" model, FOM). We have added the LPS term in order to consider equal order finite elements for velocity, pressure and temperature, respectively. Steady solutions at strongly convection-dominated regime (Rayleigh number Ra up to 10^7) are taken as snapshots.

At the reduced order level, we use a projection-based (intrusive) approach for the Boussinesq equations. The nonlinear term of the VMS-LES Smagorinsky model and the nonlinear LPS stabilization coefficients have been approximated both by an intrusive method through the Discrete Empirical Decomposition Method (DEIM) [6], giving rise to a fully intrusive ROM, and a nonintrusive/data-driven method through Radial Basis Functions (RBF) [7], giving rise to a hybrid ROM. We have performed a comparison between both methods in terms of accuracy and efficiency. Indeed, the DEIM method (intrusive) strongly depends on the specific turbulence model (and related empirical coefficients) employed at the full order level. On the contrary, the interest of the recently proposed data-driven (non-intrusive) method based on RBF is that is totally generic, regardless of the turbulence model (LES or RANS) used at the full order level.

For P2, we have used a fully non-intrusive data-driven approach based on FOM snapshots computed with Ansys FEA-CFD software. The ROM is built based on barycentric

triangulation for the selection of the parameter points and on Proper Orthogonal Decomposition for the selection of the modes. Eventually, Radial Basis Function techniques with multiquadratic kernel are used as interpolation method [10].

This combined procedure allows to construct a digital twin of the heat flow within the receiver and heat losses to ambient in the off-line stage, with very low number of degrees of freedom. The on-line stage consists in using this reduced digital twin to compute approximations of the full velocity and heat flows, as well as convective heat transfer coefficients for heat losses computations.

3. Results

For P1, the Boussinesq VMS-LES Smagorinsky ROM has been tested for 2D thermal flow in a square cavity with differentially heated vertical walls, with Rayleigh number in the strongly convection-dominated regime (up to 10^7) as physical parameter. The FOM is computed through a semi-implicit evolution approach, considering that the steady solution is reached when the error between two iterations is below 10^{-8} . We consider a uniform partition of the Rayleigh values in the range $[10^6, 10^7]$ in order to obtain 100 snapshots for the POD-ROM.

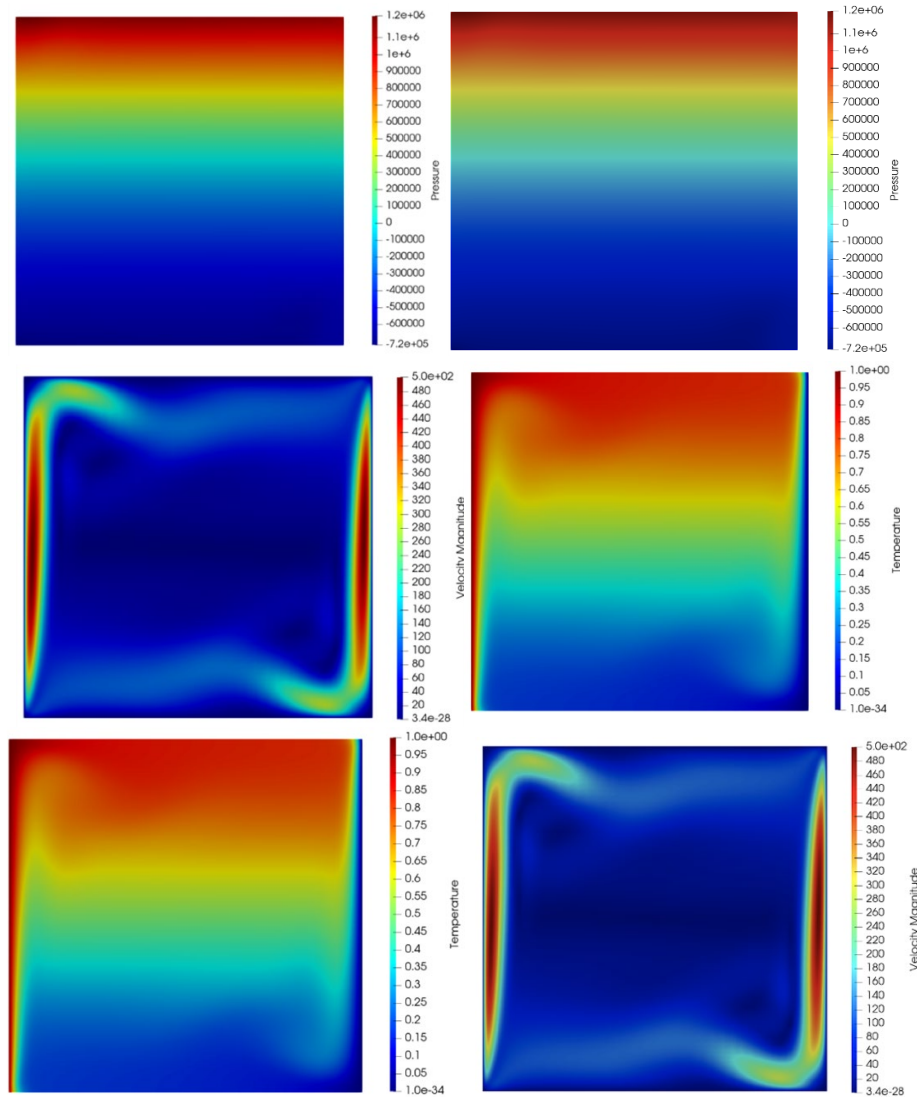


Figure 1. FOM solution (left) and ROM solution (right) of pressure, temperature and velocity magnitude (from top to bottom) for $Ra = 5173246$.

Due to the large values of Rayleigh number considered, the flow is stretched to the walls and the heat transfer is strongly driven by convection, so we decided to refine the grid towards the walls in both spatial directions using a hyperbolic tangent function.

To show the results, we used values of Rayleigh numbers in the range $[10^6, 10^7]$ but different from the trial values (snapshots). In Figure 1, we can see a comparison between the pressure, temperature, and velocity magnitude (from top to bottom) of FOM (left) and ROM (right) with $Ra = 5173246$ using data-driven (non-intrusive) method based on RBF.

Table 1. CPU time for FOM and ROM solutions, with the speedup, relative errors for velocity, temperature and pressure and the average Nusselt number with **fully intrusive** method.

Data	Ra=3245178	Ra=5173246	Ra=7986112
T_{FE} (s)	31762,3	30772,7	29968,3
T_{ROM} (s)	3,11	4,22	5,63
Speedup	10212	7292	5394
Relative L^2 error (vel)	$5,5 \times 10^{-4}$	$9,05 \times 10^{-4}$	$1,36 \times 10^{-4}$
Relative L^2 error (temp)	$5,03 \times 10^{-5}$	$8,4 \times 10^{-5}$	$1,22 \times 10^{-5}$
Relative L^2 error (press)	$1,7 \times 10^{-3}$	$2,09 \times 10^{-3}$	$3,6 \times 10^{-3}$
Average Nusselt Number - FOM	12,2041	13,8461	15,586
Average Nusselt Number - ROM	12,2032	13,8442	15,555

Table 2. CPU time for FOM and ROM solutions, with the speedup, relative errors for velocity, temperature and pressure and the average Nusselt number with **data-driven/non-intrusive** method.

Data	Ra=3245178	Ra=5173246	Ra=7986112
T_{FE} (s)	31762,3	30772,7	29968,3
T_{ROM} (s)	3,06	4,04	5,118
Speedup	10400	7617	855
Relative L^2 error (vel)	$3,22 \times 10^{-5}$	$5,1 \times 10^{-5}$	$7,08 \times 10^{-5}$
Relative L^2 error (temp)	$1,66 \times 10^{-5}$	$2,09 \times 10^{-5}$	$2,99 \times 10^{-5}$
Relative L^2 error (press)	$9,3 \times 10^{-5}$	$1,4 \times 10^{-4}$	$1,3 \times 10^{-3}$
Average Nusselt Number - FOM	12,2041	13,8461	15,586
Average Nusselt Number - ROM	12,2041	13,8461	15,586

In this benchmark, to see a comparison between the fully intrusive DEIM and the data-driven/non-intrusive method based on RBF method used to treat the nonlinear turbulence and stabilization terms, we show Tables 1 and 2, that display the speed up of the ROM, relative errors of velocity, pressure, and temperature and average Nusselt number. We remark that the speedup refers to the improvement of the computational velocity: FOM cpu time / ROM cpu time. We observe that the data-driven (non-intrusive) method presents slightly higher speed up than the fully intrusive one and the relative errors are in general comparable, being in some cases one order of magnitude lower for the data-driven (non-intrusive) method.

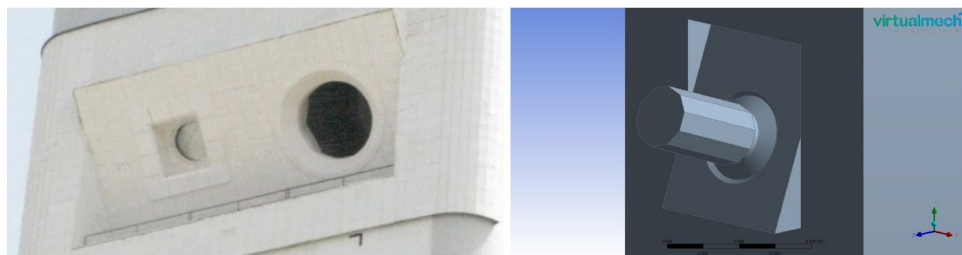


Figure 2. Left) SOLUGAS-type cavity. Right) Cavity plus ambient air control volume.

For the data-driven (or non-intrusive, see the beginning of section Reduce Order Modelling for explanation) ROM of P2, ROMs have been obtained for two parameters variation, namely: wind velocity $V = [5-30]$ m/s and cavity temperature $T = [973-1373]$ K, see Figure 2, for a SOLUGAS-like cavity receiver. The direction of the wind velocity is fixed and taken as the primary direction over the year (22.5° SE), which is quite constant. A total of 30 FOM snapshots were computed with uniform partitions on both parameters: $V = [5, 10, 15, 20, 25, 30]$ m/s and $T = [973, 1073, 1173, 1273, 1373]$ K. The comparison of computational time between FOM and fully non-intrusive ROM [10] is huge: full simulation vs off-line "interpolation", from several hours to seconds ($O(10^4)$). In order to compute the ROM approximation errors, a leave-one-out strategy has been performed, therefore in each case (set of parameters: $S_1 = [V = 5, T=1373]$, $S_2 = [V = 30, T=1373]$) 29 snapshots are used out of 30 available (one-out to compare). Figures 3 to 5 show a comparison of the FOM vs ROM with Relative Errors also computed as percentage (%). Please note that all variables (temperature, pressure and velocity have been rescaled from 1 (minimum) to 2 (maximum) to properly compute relative errors in each case. Results are shown in scaled version.

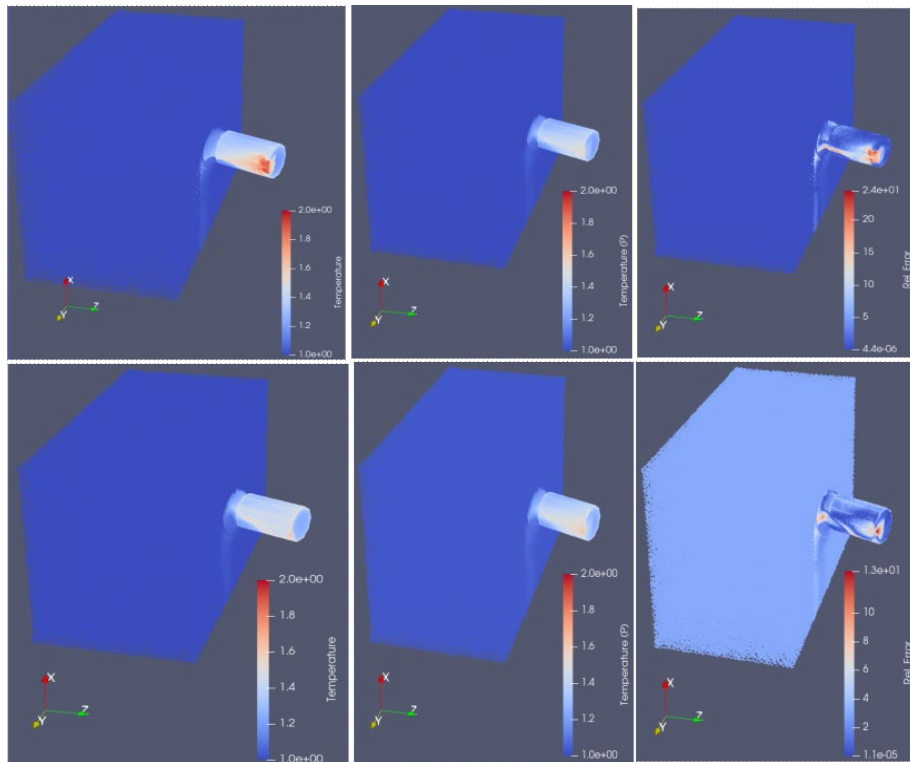


Figure 3. Temperature field: (left to right:) FOM snapshot, ROM prediction and relative errors (%) for two sets of parameter values: temperature $T=1373$ K, wind velocities $V= [5$ (top), 30 (bottom)] m/s, Leave-one-out strategy.

Figure 3 shows the scaled temperature field in the full control volume. Errors up to 24% for parameters set S_1 can be encountered in the cavity interior (13% for parameters set S_2), where higher gradients are present, however, for most of the domain errors are below 5%. These errors are expected to be maximum, since we are removing the extreme value of 1373 K from the snapshots set. For other inner points of the parameter, lower errors are expected.

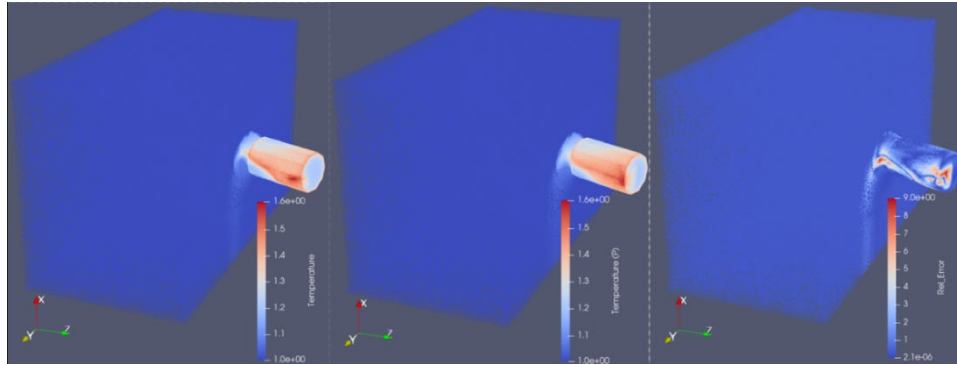


Figure 4. Temperature field: (left to right:) FOM snapshot, ROM prediction and relative errors (%) for inner parameter values set s3: temperature $T=1173$ K, wind velocity $V=20$ m/s, Leave-one-out strategy.

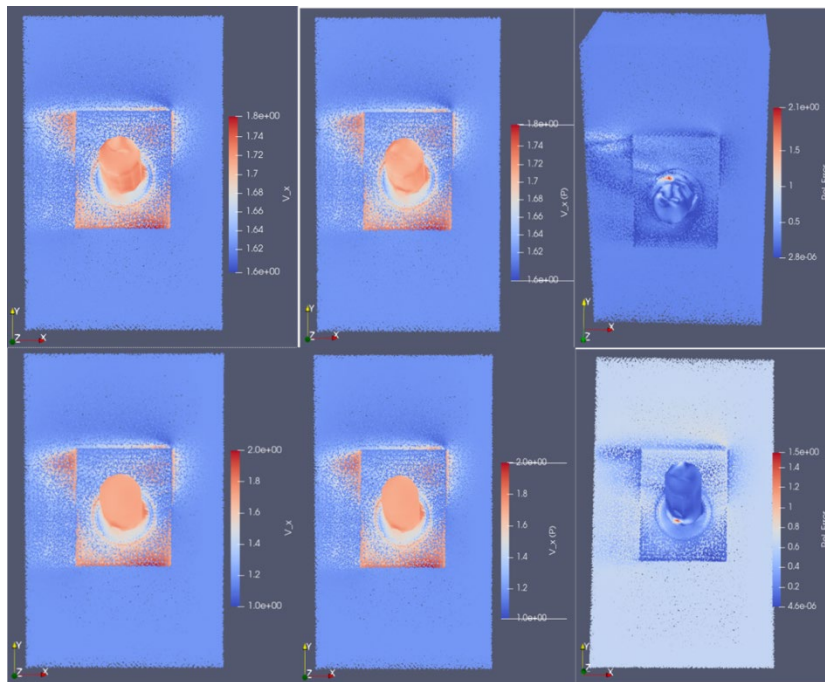


Figure 5. Pressure field: (left to right:) FOM snapshot, ROM prediction and relative errors (%) for two sets of parameter values: temperature $T=1373$ K, wind velocities $V=[5$ (top), 30 (bottom)] m/s, Leave-one-out strategy.

Eventually, the procedure is applied to the velocity field, in this case the X component, see Figure 6. For the parameters set S_1 the maximum error is 2.1% while for set S_2 we obtain 1.5%, again exceptional results. To improve the results for the temperature (Figures 3 and 4), that shows the higher errors, more FOM snapshots should be calculated. A procedure to select the best new choice of FOM snapshots is under development and it is out of the scope of the present paper. It is based on the errors computed with the leave-one-out strategy, selecting new snapshot parameter values where the errors are higher.

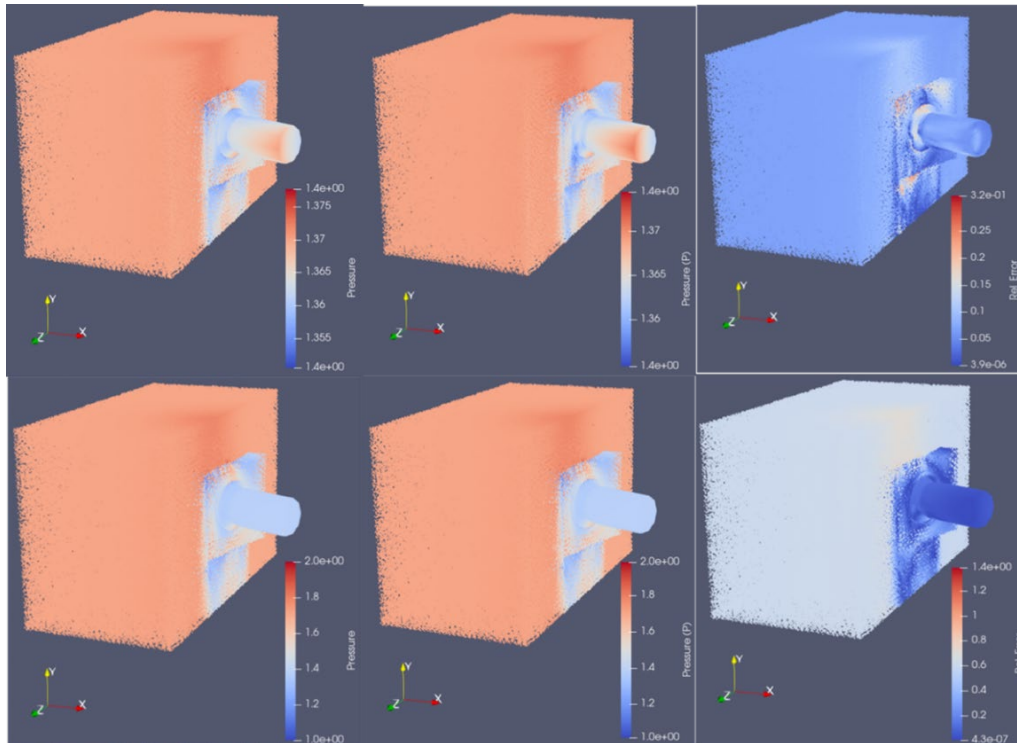


Figure 6. X component of velocity field: (left to right:) FOM snapshot, ROM prediction and relative errors (%) for two sets of parameter values: temperature $T=1373$ K, wind velocities $V=[5$ (top), 30 (bottom)] m/s, Leave-one-out strategy.

Table 3. Errors for parameter sets $S_1 = [V = 5, T=1373]$, $S_2 = [V = 30, T=1373]$.

Variable V	Re Min		Re Mean		Re Max	
	5	30	5	30	5	30
Temperature	4×10^{-6}	10^{-5}	2,23	2,89	24,4	12,9
Velocity X	3×10^{-6}	5×10^{-6}	0,30	0,42	2,1	1,5
Velocity Y	5×10^{-7}	3×10^{-6}	0,60	0,61	18,4	16,3
Velocity Z	5×10^{-6}	6×10^{-7}	0,67	0,61	18,7	29,6
Pressure	4×10^{-6}	4×10^{-7}	0,07	0,35	0,32	1,38

The errors for parameters set S_1 and S_2 are reported in Table 3. In general, non-intrusive strategy is less accurate than intrusive because we are not integrating the FOM equations in the procedure. We are only making use of the results produced by the FOM. However, but from an operative point of view, it is easier to integrate into an existing simulation environment already established, for instance in a company using commercial FOM simulation.

In conclusion, we have shown that the use of ROMs can lower the computation times several orders of magnitude in complex CFD calculation in the calculation of cavity heat losses while keeping moderate errors in the calculations.

Data availability statement

Due to the nature of the research, due to legal reasons, supporting data is not available.

Author contributions

Following CRediT taxonomy: All authors, investigation, formal analysis, JV conceptualization, methodology, JCH software, TCH conceptualization, methodology.

Competing interests

The authors declare no competing interests.

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References

1. P. Brenner, S. Grivet-Talocia, A. Quarteroni, G. Rozza, W. Schilders, L. M. Silveira. Model Order Reduction Vols. 1, 2 & 3. De Gruyter (2021) (<https://doi.org/10.1515/9783110499001>).
2. J.C. Herruzo, J. Valverde, M.A. Herrada and J. Galan-Vioque, Cavity losses estimations in CSP applications, AIP Proceedings, 1, 210007 (2018) (<https://doi.org/10.1063/1.5067209>).
3. Ballarin, F., Rebollo, T. C., Ávila, E. D., Mármol, M. G., Rozza, G. Certified Reduced Basis VMS-Smagorinsky model for natural convection flow in a cavity with variable height. Computers & Mathematics with Applications, 80(5), 973-989 (2020) (<https://doi.org/10.1016/j.camwa.2020.05.013>).
4. Rubino, S. Numerical analysis of a projection-based stabilized POD-ROM for incompressible flows. SIAM Journal on Numerical Analysis, 58(4), 2019-2058 (2020) (<https://doi.org/10.1137/19M1276686>).
5. Chaturantabut, S., & Sorensen, D. C. Discrete empirical interpolation for nonlinear model reduction. In Proceedings of the 48th IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference (pp. 4316-4321). IEEE (2009, December) (DOI: <https://doi.org/10.1109/CDC.2009.5400045>).
6. Hijazi, S., Stabile, G., Mola, A., & Rozza, G. Data-driven POD-Galerkin reduced order model for turbulent flows. Journal of Computational Physics, 416, 109513 (2020) (<https://doi.org/10.1016/j.jcp.2020.109513>).
7. Kunisch, K., & Volkwein, S. Galerkin proper orthogonal decomposition methods for parabolic problems. Numerische mathematik, 90(1), 117-148 (2001) (<https://doi.org/10.1007/s002110100282>).
8. Chacón Rebollo, T., Gómez Mármol, M., Hecht, F., Rubino, S., & Sánchez Muñoz, I. A high-order local projection stabilization method for natural convection problems. Journal of Scientific Computing, 74(2), 667-692 (2018) (<https://doi.org/10.1007/s10915-017-0469-9>).
9. Demo et al., (2018). EZyRB: Easy Reduced Basis method. Journal of Open-Source Software, 3(24), 661, <https://doi.org/10.21105/joss.00661>.