SUMO User Conference 2022
https://doi.org/10.52825/scp.v3i.119

Published: 29 Sept. 2022

Proposing a Simulation-Based Dynamic System Optimal Traffic Assignment Algorithm for SUMO: An Approximation of Marginal Travel Time

Behzad Bamdad Mehrabani[1,*][https://orcid.org/0000-0001-8585-7879], Jakob Erdmann[2][https://orcid.org/0000-0002-4195-4535], Luca Sgambi[1] and Maaike Snelder[3, 4][https://orcid.org/0000-0001-7766-2174]

[1] Université Catholique de Louvain, Louvain Research Institute for Landscape, Architecture, Built Environment (LAB), Louvain-la-Neuve, Belgium
[2] German Aerospace Center (DLR), Berlin, Germany
[3] Delft University of Technology, Civil Engineering Faculty, Delft, Netherlands
[4] Netherlands Organization for Applied Scientific Research (TNO), Hague, Netherlands

Abstract

User equilibrium (UE) and system optimal (SO) are among the essential principles for solving the traffic assignment problem. Many studies have been performed on solving the UE and SO traffic assignment problem; however, the majority of them are either static (which can lead to inaccurate predictions due to long aggregation intervals) or analytical (which is computationally expensive for large-scale networks). Besides, most of the well-known micro/ meso traffic simulators, do not provide a SO solution of the traffic assignment problem. To this end, this study proposes a new simulation-based dynamic system optimal (SB-DSO) traffic assignment algorithm for the SUMO simulator, which can be applied on large-scale networks. A new swapping/convergence algorithm, which is based on the logit route choice model, is presented in this study. This swapping algorithm is compared with the Method of Successive Average (MSA) which is very common in the literature. Also, a surrogate model of marginal travel time was implemented in the proposed algorithm, which was tested on real and abstract road networks (both on micro and meso scales). The results indicate that the proposed swapping algorithm has better performance than the classical swapping algorithms (e.g. MSA). Furthermore, a comparison was made between the proposed SB-DSO and the current simulation-based dynamic user equilibrium (SB-DUE) traffic assignment algorithm in SUMO. This proposed algorithm helps researchers to better understand the

* Corresponding author: behzad.bamdad@uclouvain.be

impacts of vehicles that may follow SO routines in future (e.g., Connected and Autonomous Vehicles (CAVs)).

1 Introduction

One of the most critical factors in the transportation planning process is solving the traffic assignment problem (Bamdad Mehrabani et al., 2021). Traffic assignment determines the routes that are used by vehicles based on certain behavioral principles, such as, for example, each vehicle seeks to minimize its own travel time. Many of the current traffic assignment algorithms are based on the two behavioral principles of Wardrop (Wardrop, 1952): (1) Wardrop's first principle: under user equilibrium (UE) conditions, no vehicle can unilaterally reduce its travel time by shifting to another route; (2) Wardrop's second principle: under system optimal (SO) conditions, traffic should be arranged in congested networks such that the average (or total) travel time is minimized. The first principle (UE) assumes that each vehicle attempts to minimize its own travel time (selfish routing). In contrast, in the second principle (SO), it is considered that each vehicle selects a route that minimizes not only its own travel time but also the entire network's travel time.

Traffic assignment methods can be broadly classified into two categories: 1) static traffic assignment (STA) and 2) dynamic traffic assignment (DTA) (Saw et al., 2015; Tsanakas, 2019). In STA, the traffic demand is static with respect to time and is typically used for strategic transportation planning. In DTA, the traffic demand is not static and varies over time, and the arrival time at a link is different from the departure time. Although the computational expenses of DTA are higher than those of STA, DTA attracts researchers owing to the several limitations of STA, for example, 1) limitations of static models because of the use of volume delay functions (such as no overtaking effects and no representation of the phenomenon of congestion spillback), 2) limitations in modeling of signal synchronization, 3) limitations in modeling of lane-based effects (such as high-occupancy vehicle lanes), and 4) limitations in modeling intelligent transportation system-related applications, such as traveler information systems (Chiu et al., 2011).

Different models exist in the literature to solve the DTA. The most important models are listed in Table 1.

Table 1: Different approaches of DTA

<table>
<tr><th>Model</th><th>Approach</th></tr>
<tr><td rowspan="3">Analytical Based Model</td><td>Mathematical Programming</td></tr>
<tr><td>Optimal Control Formulations</td></tr>
<tr><td>Variational Inequality-Based</td></tr>
<tr><td rowspan="2">Simulation-based Model</td><td>Micro Simulation</td></tr>
<tr><td>Meso Simulation</td></tr>
</table>

Analytical models of DTA use analytical formulations to predict the propagation of traffic in a network (network loading). Traffic propagation models, which are used in analytical models, are typically based on extensions of the Lighthill–Whitham–Richards (LWR) model (Lighthill & Whitham, 1955; Richards, 1956). LWR is a macroscopic one-dimensional traffic model that uses traffic density and speed for traffic flow propagation (Li, 2016). The traffic flow propagation during dynamic network modeling can be based on the cell transmission model (Ziliaskopoulos, 2000) or link transmission model (Yperman, 2007). Although the mathematical closed-form is available for the analytical solution algorithm (thus, they are highly accurate), in practice, they cannot model certain phenomena (such as individual vehicles and vehicle interaction) in detail due to their macro-scale nature. Also, applying analytical assignment problems to large-scale networks may be highly timeconsuming and complex to solve (Gawron, 1998). Simulation-based traffic assignment models use a traffic simulator to replicate the traffic flow dynamics (propagation) and spatio-temporal interactions (e.g., vehicle movements), which are based on micro/meso traffic flow simulation models (Saw et al., 2015). In addition, a traffic simulator is used as part of the search process to determine the optimal solution (Peeta & Ziliaskopoulos, 2001). They typically conduct several iterations to obtain optimum values (no closed-form is available). Compared to the analytical model, the simulation-based model appears to be more practical because of its ability to explain traffic flow propagation in more detail.

In the near future, several road users (e.g., connected and autonomous vehicles (CAVs)) are expected to follow Wardrop's second principle (SO) routines (Bagloee et al., 2017; Mansourianfar et al., 2021; J. Wang et al., 2019). It is important to provide a simulation-based dynamic system optimal (SB-DSO) traffic assignment model as a powerful tool to evaluate the impacts of such users. The dynamic system optimal (DSO) traffic assignment problem has been thoroughly studied in the literature. However, most of these works are based on an analytical model (e.g., (J. Liu et al., 2020; Ngoduy et al., 2021; Shen et al., 2006; Shen & Zhang, 2009; Tajtehranifard et al., 2018; Wie et al., 1990)) and very few studies utilize a simulation-based model which either are only microscopic or only mesoscopic (Ameli et al., 2020a; Hu et al., 2018; Mansourianfar et al., 2021; Peeta & Mahmassani, 1995; Sbayti et al., 2007; Yang & Jayakrishnan, 2012). Nevertheless, previous studies have demonstrated the superior performance of the simulation-based dynamic traffic assignment (SB-DTA) model (Ameli et al., 2020a), but many open-source (e.g., simulation of urban mobility (SUMO)) and commercial (e.g., Aimsun) traffic simulation software products do not provide SB-DSO traffic assignment algorithms. Therefore, this study contributes to the literature by proposing a SB-DSO traffic assignment algorithm based on a new swapping/convergence method that implements a logit route choice model for SUMO. The proposed algorithm can be applied on both micro and meso models of traffic flow which replaces the travel time of a link with a surrogate model of marginal travel times (MTT) to shift from DUE to DSO. To better understand the performance of the proposed algorithm, three case studies were conducted. A comparison was made between the proposed SB-DSO and the current simulation-based dynamic user equilibrium (SB-DUE) traffic assignment in SUMO. The remainder of this paper is organized as follows. The notations and abbreviations used in this paper are presented in section 2. In section 3, the literature is reviewed, followed by the research methodology (SB-DSO framework) in section 4. The proposed algorithm is applied to three case studies in section 5, and the conclusions are presented in Section 6.

2 Notations and Abbreviations

The list of all abbreviations used in this paper is included in Table 2. The notations used to present the proposed SB-DSO solution algorithm are listed in Table 3.

Table 2: The list of abbreviations in the paper

Abbreviation	Meaning	Abbreviation	Meaning
ADT	Average Distance Travelled	MSWA	Method of Successive Weighted Average
AS	Average Speed	MSAR	Method of Successive Average Ranking
ATL	Average Time Loss	MTT	Marginal Travel Time
CAV	Connected and Autonomous Vehicles	O-D	Origin-Destination pairs
DSO	Dynamic System Optimal	SA	Simulated Annealing
DTA	Dynamic Traffic Assignment	SB-DSO	Simulation-Based Dynamic System Optimal
DUE	Dynamic User Equilibrium	SB-DUE	Simulation-Based Dynamic User Equilibrium
DNL	Dynamic Network Loading	STA	Static Traffic Assignment
GA	Genetic Algorithm	SO	System Optimal
LWR	Lighthill–Whitham–Richards	TTT	Total Travel Time
MSA	Method of Successive Average	UE	User Equilibrium

Table 3: Notations

Indices	
c	Index for travel times (cost)
f	Index for traffic flow
i	Index for iteration steps
k	Index for path
Sets	
$G(V, A)$	traffic network
A	set of links ($a \in A$)
V	set of nodes ($v \in V$)
$J(R, S)$	set of vehicles ($j \in J$)
R	set of origin nodes ($r \in R$)
S	set of all destination nodes ($s \in S$)
I	set of simulation iterations ($i \in I$)
$P_{j,i}^{r-s}$	set of alternative paths for vehicle j in iteration i, travel from origin r to destination s
Variables, parameters, and elements	
c_a'	empty network travel time
c_a^i	travel time of link a in iteration i
$\bar{c}_a^i$	marginal travel time of link a in iteration i
C_k^i	travel time of path k in iteration i
$p_{j,i}^{r-s}$	selected path for vehicle j in iteration i, travel from origin r to destination s
$p_{j,i}^{*,r-s}$	adjusted selected path for vehicle j in iteration i, travel from origin r to destination s
$pr_{k,j}^i$	probability of selecting path k by vehicle j in iteration i
RSD_n^i	relative standard deviation of average travel time in the last n elements of i^{th} iteration
$av_{i'}$	the average travel time of the entire network in iteration i'

3 Literature Review

The Simulation-based DTA problem is split into two parts: 1- a simulation-based dynamic network loading (DNL) model and 2- an algorithm for finding the equilibrium solution (Ameli et al., 2020b). The DNL process explains the traffic flow dynamics and determines “how flows propagate with time through the network along the selected paths” (Jaume Barceló, 2010). A traffic simulator is used as the dynamic network-loading model in the simulation-based solution of the DTA problem. Whereas the second part determines the path used by the vehicles and the proportion of demand at each instant in time allocated to this determined path.

To find the equilibrium solution in simulation-based methods, usually, an iterative scheme is employed. These iterative methods start from an initial solution and update the path flow distribution for each iteration based on a path swapping algorithm. The reassignment process of vehicles in each iteration confirms whether the algorithm is in a descent direction or not. In other words, the algorithm forces vehicles at each iteration to follow a more efficient path than the previous iteration. Also, at the end of each iteration, a convergence criterion (or error) is calculated to check the algorithm's termination.

This approach was initially developed by Mahmassani and Peeta (Mahmassani & Peeta, 1993, 1995) and Peeta and Mahmassani (Peeta & Mahmassani, 1995). They incorporated a mesoscopic traffic simulator, DYNASMART (Jayakrishnan et al., 1994) (as the DNL model), in an iterative search solution framework to calculate the (marginal) travel times under the assumption of information availability for advanced traveler information system operations. The Method of Successive Averages (MSA) is used as the path swapping algorithm, and the convergence criterion is based on the differences in the number of vehicles assigned to various paths over successive iteration. The local approximation of MTT is evaluated by summing the link MTTs along the path according to the time-dependent link traversal times.

There are many studies in the literature based on the solution algorithm of Peeta and Mahmassani (Peeta & Mahmassani, 1995). For instance, Sbayti et al. (Sbayti et al., 2007) used MSA to solve the SB-DTA (with both DUE and DSO) in large-scale networks. They presented two new implementation techniques to address the disadvantages of MSA. Similar to Peeta and Mahmassani, the DYNASMART simulator was used to calculate the time-dependent link travel times, turn penalties, and link marginals. In addition, Yang and Jayakrishnan (Yang & Jayakrishnan, 2012) attempted to address the disadvantages of MSA in SB-DTA problems by implementing a gradient projection method. This study used the PARAMICS software in the DNL process. However, the travel times (generated by PARAMICS) are not directly fed into the route assignment procedure (the proposed gradient projection algorithm). As the proposed gradient projection algorithm requires an analytical function that represents link costs as traffic loads, link performance functions are used to calculate the path travel times in the path assignment process. An SB-DTA procedure was developed by Hu et al. (Hu et al., 2018) using a dynamic traffic simulator called DynaTAIWAN. The dynamic traffic simulator is used to simulate traffic flow distributions based on vehicle properties and routes (calculating the link travel time in each iteration). Four different vehicle class types (car, bus, motorcycle, and truck) and four different behavioral rules, including the pre-specified-path driver, UE driver, SO driver, and real-time information driver, are considered in the solution procedure. The MSA is applied to update the vehicles' path in each iteration. In a similar study, Mansourianfar et al. (Mansourianfar et al., 2021) developed an SB-DTA algorithm for mixed UE and SO users. They used the Aimsun traffic simulator instead of DynaTAIWAN to calculate the travel times. To address the shortages of MSA, they examined the method of successive weighted average (MSWA), which gives higher weights to later auxiliary flow patterns. This study proposes a new hybrid convergence criterion to find the mixed equilibrium solution. Another study that tried to overcome MSA's drawbacks is Ameli et al. (Ameli et al., 2020b). They study two new solution methods for the SB-DUE problem: a new extension of simulated annealing (SA) and an adapted genetic algorithm (GA). A comparison is made between the proposed meta-heuristic algorithms (SA and GA) and the classic methods (MSA, MSA ranking (MSAR)), and a gap-based algorithm). The results show that meta-heuristic algorithms dominate classical methods. However, it should be pointed out that all of the proposed meta-heuristic algorithm uses MSA as part of their solution. Also, this study implements a microscopic traffic simulator named Symuvia, which does not have meso modeling features for a trip-based dynamic simulation. Adopting the MATSIM software, Lämmel and Flötteröd (Lämmel & Flötteröd, 2009) developed an agent-based microsimulation DTA model. They replaced the travel time (based on which agents evaluate their routes) with the MTT to achieve SO. The results indicate that a simulation-based system leads to an acceptable approximation of the SO mathematical solution. There is also another group of studies on sustainable optimal DTAs (Chen et al., 2021; Lu et al., 2016). For instance, Lu et al. (Lu et al., 2016) solved an eco-system optimal DTA problem based on analytical and simulation-based models. Their study aimed to determine the SO ecological routes that minimize total vehicular emissions. The proposed simulation-based model combines macroscopic and microscopic traffic descriptions (mesoscopic) based on Newell's (Newell, 2002) simplified kinematic wave model and a simplified car-following model. In addition, this study introduces a novel approximation for path marginal emissions based on path MTT. Although the numerical examples of this study demonstrate the effectiveness of the model, it adopted a simplified car-following model (Newell) and not the commonly used car-following models, such as Krauß et al. (Krauß et al., 1997) and Gipps (Gipps, 1981).

As mentioned earlier, the traffic simulator software can be regarded as DNL of DTA (W. Wang et al., 2018); therefore, the available DTA solution methods in the most widely used and well-known micro/meso traffic simulator packages are presented in Table 4.

Table 4: Micro/Meso traffic simulation packages and their available DTA solution methods

<table>
<tr><th>Simulator</th><th>Developer</th><th>Scale</th><th>Available DTA Methods</th></tr>
<tr><td>Aimsun</td><td>J Barceló & Ferrer, 1997</td><td>Micro/Meso</td><td>Stochastic Route Choice</td></tr>
<tr><td>CONTRAM</td><td>Taylor, 2003</td><td>Meso</td><td rowspan="5">DUE</td></tr>
<tr><td>CORSIM</td><td>US-DOT, 1995</td><td>Micro</td></tr>
<tr><td>DRACULA</td><td>R. Liu, 2010</td><td>Micro</td></tr>
<tr><td>DTALite</td><td>Zhou and Taylor, 2014</td><td>Agent-based</td></tr>
<tr><td>Dynameq</td><td>Mahut, 2001</td><td>Micro</td></tr>
<tr><td>DynaMIT</td><td>Ben-Akiva et al., 1997</td><td>Meso</td><td>Converge to observed flows</td></tr>
<tr><td rowspan="3">DynaSMART</td><td rowspan="3">Jayakrishnan et al., 1994</td><td rowspan="3">Meso</td><td>Instantaneous information</td></tr>
<tr><td>Predictive information</td></tr>
<tr><td>DSO</td></tr>
<tr><td>DynusT</td><td>Y. C. Chiu et al., 2011</td><td>Meso</td><td rowspan="5">DUE</td></tr>
<tr><td>INTEGRATION</td><td>Van Aerde et al., 1996</td><td>Micro/Meso</td></tr>
<tr><td>MATSIM</td><td>Dobler and Nagel, 2016</td><td>Agent-based</td></tr>
<tr><td>PARAMICS</td><td>Smith et al., 1995</td><td>Micro</td></tr>
<tr><td rowspan="2">PTV Vissim</td><td rowspan="2">Fellendorf, 1996</td><td rowspan="2">Micro/Meso</td></tr>
<tr><td>Stochastic Assignment</td></tr>
<tr><td>SUMO</td><td>Lopez et al., 2018</td><td>Micro/Meso</td><td>(Stochastic) DUE</td></tr>
</table>

Table 4 and the literature review revealed that, thus far, no study has proposed an SB-DSO algorithm using common traffic simulators (that can address the traffic flow propagation during dynamic network modeling with high accuracy in both micro and mesoscale simulations). Therefore, this study developed a new SB-DSO algorithm by implementing the SUMO traffic simulator, in which a new swapping algorithm (based on logit route choice model) and a new convergence criterion are incorporated.

4 Methodology

The simulation-based solution of the DTA problem does not include any closed-form analytical solution and typically relies on an iterative procedure. SUMO traffic simulator provides several tools and options for solving traffic assignment and route choice problem of vehicles (simulation-based approach). *duaIterate.py* (DLR, 2021) is the solution tool for the SB-DUE problem for micro and meso levels in SUMO. This study proposes a new solution framework for the SB-DSO problem based on *duaIterate.py*. The main difference between the proposed algorithm and the current algorithm of *duaIterate.py* is that the proposed algorithm replicates the travel time by a surrogate model of MTT. Also, a new swapping algorithm and convergence criterion are presented and tested against classical methods. Figure 1 illustrates the proposed solution framework for the SB-DSO problem. As shown in Figure 1, the framework consists of two parts: path selection procedure and DNL. For the path selection procedure, *duarouter*, which is an available algorithm in SUMO for the calculation of the shortest path, is used. For the DNL procedure, the SUMO traffic simulator is used.

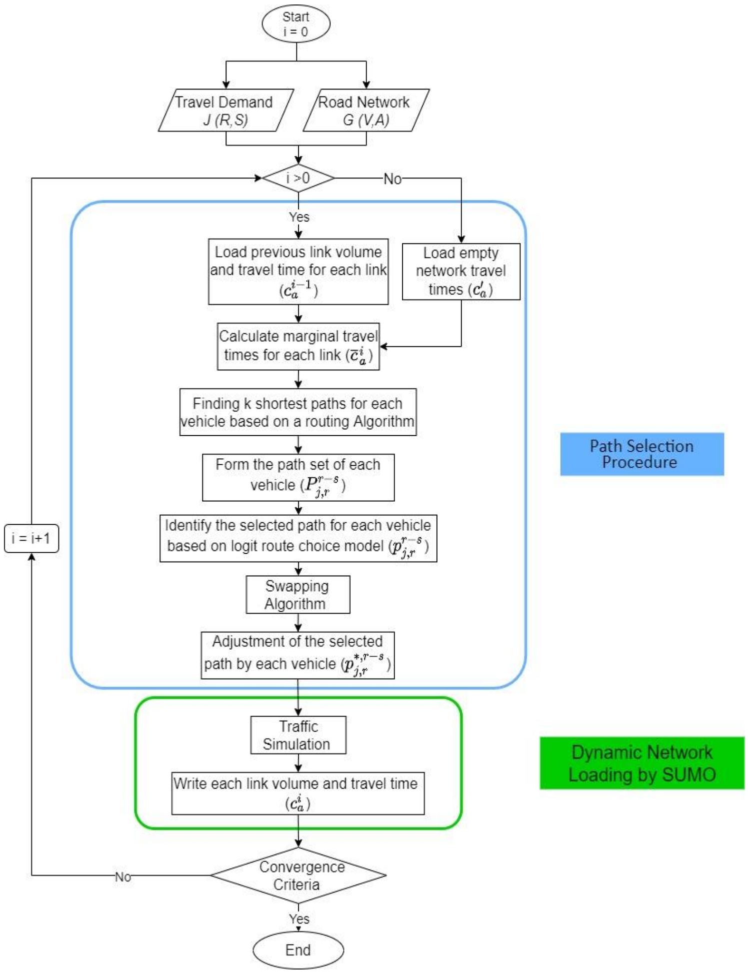


Figure 1: Framework of the SB-DSO traffic assignment

Consider $G(V, A)$ as the directed traffic network, which includes a set of links A $(a \in A)$ and a set of nodes V $(v \in V)$. $J(R, S)$ represents the set of vehicles (demand file: usually imported to sumo by $Trips.XML$ file) between the origin and destination pairs where R $(r \in R)$ and S $(s \in S)$ denote the set of all origin nodes and the set of all destination notes, respectively. Hence, j^{r-s} is a vehicle that travels from origin r to destination s. The problem is stated as the assignment of $J(R, S)$ to $G(V, A)$. The equilibrium condition is computed by iteratively calculating the shortest routes and travel times. At each simulation iteration $i \in I$, first, a routing algorithm (Dijkstra, astar, Contraction Hierarchies (CH), or CHWrapper) is applied by *duarouter* to the road network to determine the set of alternatives paths, $P_{j,i}^{r-s}$, for each vehicle j^{r-s} (at each iteration, a new alternative path set is generated for each vehicle). The k-shortest paths are calculated using the previous simulation corresponding link MTT, $\bar{c}_a{}^{i-1}$. Next, a route choice model (Gawron, Logit, or Lohse) is applied to the set of alternative paths, $P_{j,i}^{r-s}$, to select a path, $p_{j,i}^{r-s}$ $(p_{j,i}^{r-s} \in P_{j,i}^{r-s})$. Then, a swapping algorithm is implemented to reassign a fraction of vehicles at each iteration (not all vehicles change their route necessary during successive iterations), ensuring the improvement of the selected path over iterations. Finally, the adjusted selected path of each vehicle, $p_{j,i}^{*,r-s}$ (known as *trips.rou.XML* file in SUMO), is sent to SUMO to perform the traffic simulation and consequently calculates the current travel time of each link, c_a^i (known as *edgedata* output in SUMO). The travel times written in this step, c_a^i, are used as an input in the next iteration step. By performing such a process iteratively, the total travel time (TTT) is minimized (SO condition).

4.1 Route Choice Model

In SUMO, it is possible to choose different route choice models among available alternatives, which are Gawron, Logit, or Lohse. In this study, the logit model is selected as the route choice model. Thus, the proposed algorithm computes the stochastic SB-DSO solution. The logit model is applied to each vehicle's set of alternative routes, $P_{j,i}^{r-s}$, in which the k-shortest paths for the subject vehicle are available. The travel times are considered as the cost for each alternative path. The travel time of each path is equal to the sum of the travel times of the corresponding links from the previous simulation. The logit model formulation is as follows

$$pr_{k,j}^i = \frac{\exp(-\theta C_k^i)}{\sum_1^k \exp(-\theta C_k^i)} \tag{1}$$

$$C_k^i = \sum_{a \in A} \delta_{a,k}^i \, c_a^i \tag{2}$$

$$\delta_{a,p}^i = \begin{cases} 1 \; if \; link \; a \; is \; on \; path \; k \\ \quad 0 \; otherwise \end{cases} \tag{3}$$

where $Pr_{k,j}^i$ is the probability of selecting path k by vehicle j in iteration i ; C_k^i is the travel time (cost) of path k in iteration i; and θ is the logit model scale parameter. Given the multiple alternative routes with slightly different travel times, it may be reasonable to select a route other than the strictly shortest route (to avoid congestion on that route). Hence, the scale parameter θ assigns a probability for each route alternative. With a high value of theta, logit always selects the route with the least travel time, whereas with a low value of theta, logit selects all the routes with almost equal probability.

It should be mentioned that although this study works on the stochastic solution of the traffic assignment problem, it is possible to reach the deterministic solution by the same proposed algorithm (by replacing the route choice model with All-or-Nothing assignment) in SUMO.

4.2 Calculation of Marginal Travel Times

Previous studies have proven that the SO condition can be achieved by replacing the path travel time with the path MTT (Hu et al., 2018; Mansourianfar et al., 2021; Patriksson, 2015; Rahman et al., 2015). There are two ways to calculate the path MTT (Ameli et al., 2020a; Mansourianfar et al., 2021): 1) global approximation, which represents the changes in the total system travel time caused by an additional vehicle that is added to the path at a certain time interval, and 2) local approximation (Ghali & Smith, 1995; Peeta & Mahmassani, 1995), which represents the changes in the path travel time caused by an additional vehicle that is added to the route at a certain time interval. This approach considers the path MTT as a summation of the corresponding link MTTs. Although it has been proven that such a local approximation may lead to overestimation of the path MTT (Qian et al., 2012; Shen et al., 2007) it is a practical approximation in large-scale networks (Mansourianfar et al., 2021). On the other hand, because the global approximation of MTT is computationally expensive and is not practical for large-scale DTA, this study implements the local approximation of MTT. To achieve the local approximation of path MTT, first, the MTT of each link should be calculated; then, the MTTs of the corresponding links in each path are summed up. Numerous formulations exist in the literature to approximate the MTT of the links; however, it may be inappropriate to compare these formulations numerically as they use their own traffic flow propagation method and assumptions (Doan & Ukkusuri, 2015; Qian et al., 2012; Zhang & Qian, 2020). In addition, the traffic simulators (as the DNL model) used in each study are different. Sheffi (Sheffi, 1985) formulated the link MTT as follows and defined it as the "marginal contribution of an additional traveler on the a^{th} link to the TTT on this link." In other words, MTT is the derivative of the travel time with respect to flow:

$$\bar{c}_a(f_a) = c_a(f_a) + f_a \frac{dc_a(f_a)}{df_a} \tag{4}$$

This formulation is the sum of the two components. The first component, $c_a(f_a)$, is the travel time experienced by the additional traveler when the total link flow is f_a. This component can be explained by the average travel time on link a. The second component is the multiplication of $\frac{dc_a(f_a)}{df_a}$, the additional travel time burden that the additional traveler inflicts on each of the other travelers, by the number of travelers that already exist on the link (f_a). Therefore, the effect of one additional user on all the other travelers is considered by the second component.

Given that SUMO provides the average travel time of each link, it is not possible to calculate the additional travel time that one vehicle inflicts on the link. An alternative approach to compute the link MTT is to calculate the average travel time in successive iterations (with a different number of vehicles assigned to each link in each iteration) and compute the difference in link average travel time. Using this method, the average inflicted additional travel time on the link can be calculated. Therefore, in this study, we developed a surrogate model of MTT to achieve SO as follows:

$$\bar{c}_a^i = c_a^{i-1} + f_a^{i-1} \frac{c_a^{i-1} - c_a^{i-2}}{f_a^{i-1} - f_a^{i-2}} \tag{5}$$

where $\bar{c}_a^i$ is the surrogate MTT of link a at simulation step i; c_a^{i-1} and c_a^{i-2} are, respectively, the travel time (cost) of link a at simulation steps $i-1$ and $i-2$; and f_a^{i-1} and f_a^{i-2} are, respectively, the traffic flow of link a at simulation steps $i-1$ and $i-2$. The first term of this equation represents the average travel time of link a and the second term represents the average inflicted additional travel time on the link. In other words, the second term of this model indicates the extent to which adding one vehicle to a link leads to an increase in the travel time of the vehicles that are already in the link. In this way, instead of feeding only the average travel time that each vehicle would experience along a route, the additional cost that it imposes on the total travel time by selecting the route is added. Hence, the additional travel time that other vehicles must "pay" (on average) is addressed if the subject vehicle selects that route.

4.3 Swapping Algorithm

Most studies that implement simulation-based traffic assignment methods employ a swapping algorithm to reach the optimum value and avoid oscillating. The core idea of swapping algorithms is that not all vehicles should necessarily change their path in each iteration; instead, only a fraction of vehicles is in the reassignment process. Thus, a proper direction for the next iteration is obtained. Most of the previous studies apply the MSA as their swapping algorithm. The conventional MSA is calculated as:

$$f^i = \left(\frac{i}{i+1}\right) f^{i-1} + \left(\frac{1}{i+1}\right) y^i \tag{6}$$

In which f^i is the path flow distribution of iteration i, y^i is the auxiliary path assignments obtained by all-or-nothing assignment, and i is the number of iterations. In the primary iterations, the value of step size is too large, thus the travel time of vehicles does not reduce after several iterations. While in the last iterations the value of step size is too small, which leads to slow convergence speed. Therefore, previous studies provided several heuristic algorithms (Ameli et al., 2020b) or extensions of MSA (like Method of Successive Weighted Averages (MSWA) (H. X. Liu et al., 2009) or MSA Ranking (MSAR) (Sbayti et al., 2007)) to address the disadvantages of MSA. However, most of the previous studies implement these extensions in combination with deterministic traffic assignment (all-or-nothing assignment which is highly sensitive to the small changes in traffic flow).

This study proposes a new swapping algorithm for stochastic traffic assignment problems which is based on the logit route choice model. The new swapping algorithm is as follows:

$$p_{j,i}^{*,r-s} = \begin{cases} p_{j,i}^{r-s} & if\ x \geq \rho_i \\ p_{j,i-1}^{*,r-s} & if\ x < \rho_i \end{cases} \tag{7}$$

Where $p_{j,i}^{*,r-s}$ is the adjusted selected path by vehicle j in iteration i; $p_{j,i}^{r-s}$ is the selected path by vehicle j in iteration i from current logit model; $p_{j,i-1}^{*,r-s}$ the adjusted selected path by vehicle j in iteration $i-1$; x is a random variable between 0 and 1; and ρ_i is the sequence of step size in each iteration which can be considered as the probability of keeping the previous adjusted selected path. In this study ρ_i is predetermined $\rho_i = \frac{i}{\gamma}$; where i is the iteration number, and γ is a scale parameter. γ is a real number that determines the speed of convergence. With a low value of γ, the speed of the convergence is fast, but few alternative paths are tested by each vehicle. On the other hand, with a high value of γ, the convergence speed is slow, while several alternative paths (which are available in the path set) will be tested by each vehicle. Therefore, for stochastic assignments, it can be argued that higher values of γ are preferable. However, for large and medium scale networks, it is computationally expensive to wait for high number of iterations. In this study the value of γ is set to 100 and 50 for small scale and medium/large scale networks, respectively. This swapping algorithm prevents some vehicles, in successive iterations, from changing their routes, allowing them to follow the path they have chosen in the previous iteration. This ensures that the algorithm leads to the improvement of the selected path by each vehicle. For convenience of description, we name this swapping algorithm as "PSwap" (Probabilistic Swapping). PSwap is tested against the revised version of MSA in which the auxiliary path is obtained by the logit route choice model. In order to better understand the differences between two swapping algorithms, figure 2 shows the (maximum) fraction of vehicles that could change their routes per iteration.

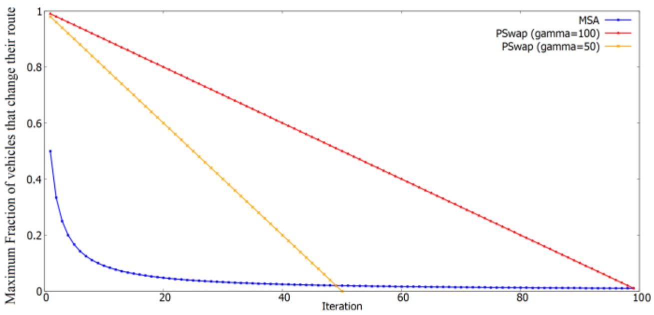


Figure 2: Maximum Fraction of vehicles that change their route per iteration

4.4 Convergence Criterion

Many studies in the past have provided different convergence criteria for the SB-DTA algorithm termination. As no closed-form is available for the simulation-based solutions, it is impossible to mathematically prove the algorithm's convergence. Therefore, all of the convergence criteria only provide some point where the algorithm can be terminated. A common approach in previous studies for the calculation of convergence criterion (error) is to calculate the maximum difference between the route flows of two iterations (e.g. (Peeta & Mahmassani, 1995)). However, previous studies pointed out that, this criterion does not guarantee the equilibrium condition since applying any swapping algorithm yield smaller route flow changes in successive iterations by default (Taale & Pel, 2015). Another common approach presented by previous studies is to calculate the relative gap between the current travel times of vehicles in each iteration and the least experienced travel times (Ameli et al., 2020b; Mansourianfar et al., 2021). However, the value to which the gap converges is not known beforehand, thus it is difficult to determine if convergence is reached (Taale & Pel, 2015).

As in equilibrium condition, no driver can unilaterally reduce his/her travel time by shifting to another route, when the standard deviation of average travel times between successive iterations is low, it implies that all remaining alternatives have "almost" the same travel time so the actual DUE criterion (or it's DSO equivalent) are basically fulfilled. Therefore, this study considers the relative standard deviation of average travel time (in the entire network) as convergence criteria:

$$RSD_n^i = \frac{\sqrt{\frac{1}{n}\sum_{i'=(i-n)+1}^{i}(av_{i'} - \overline{av}_n^i)^2}}{\overline{av}_n} \tag{8}$$

$$\overline{av}_n = \frac{\sum_{i'}^{i} av_{i'}}{n} \tag{9}$$

Where RSD_n^i is the relative standard deviation of average travel time in the last n elements of i^{th} iteration; $av_{i'}$ is the average travel time of the entire network in iteration i'; and $\overline{av}_n$ is the mean of $av_{i'}$ in the last n iterations. This criterion evaluates the dispersion of average travel times in last n iterations. Low values of RSD_n^i represent that average travel time does not vary over successive iterations; thus, a decent point for termination is found. The proposed algorithm is considered being converged (after the minimum number of 10 iterations) if the value of RSD_n^i, become constant and less than ε (fixed at 0.05 for micro simulations and 0.005 for meso simulations).

5 Test Networks

The proposed algorithm was studied in three different networks (Figure 3): (a) small-size Braess like network, (b) medium-size abstract Random network, and (c) large size Sioux Falls network.

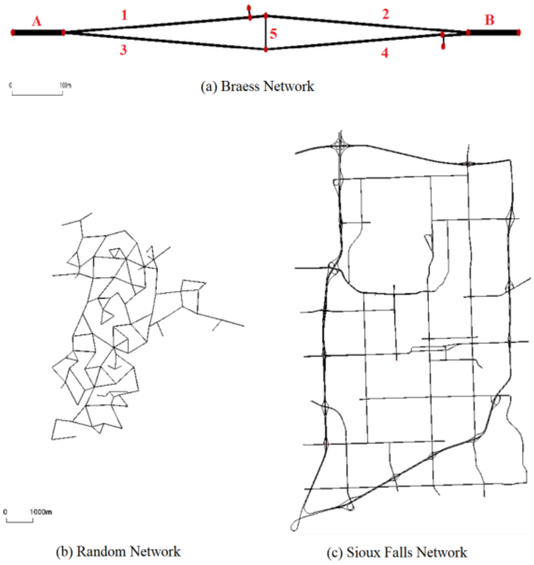


Figure 3: Test Networks

The Braess and Sioux Falls networks are commonly used as benchmarks in the literature. A random network was implemented to evaluate the performance of the algorithm in random unknown cases. In the following sections, the results of traffic simulations for the test networks are presented. For each network, four scenarios are simulated: (1) SB-DSO-PSwap, (2) SB-DSO-MSA, (3) SB-DUE-PSwap, (4) SB-DUE-MSA. The TTT in different iterations of each scenario is given first. The TTT is considered as the sum of travel times of all vehicles in the network and the waiting times of the vehicles which cannot insert into the network during the simulation. Then, traffic-related measures (including TTT (s), average speed (AS) (m/s), the average distance traveled (ADT) (m), and average time loss (ATL) (s)) of each scenario are evaluated. Finally, to have a better understanding of the differences between the proposed SB-DSO and the current SB-DUE, the traffic volume in the best iterations is illustrated.

5.1 Braess Network

In this study, a network similar to the Braess network (Figure 2 (a)) was simulated in four scenarios to evaluate the changes in TTT when the vehicles follow the proposed SB-DSO and DB-DUE. The microsimulation is performed with the demand of 1000 vehicles/h, departs from node A to destination B. Three routes are available for each vehicle. Each route includes a high-speed link (link 1 or 4), a low-speed link (link 2 or 3), and/or a high-speed shortcut (link 5). To analyze the behavior of the proposed algorithm (SB-DSO), the convergence patterns of the algorithms are presented in Figure 3 (value of TTT in successive iterations), while figure 4 shows the convergence pattern of the SB-DUE algorithm. For the sake of comparison, the fraction of vehicles that change their route per iteration is illustrated in figure 6.

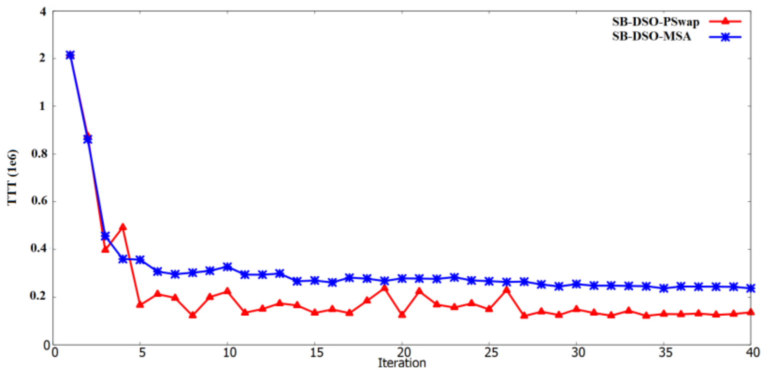


Figure 4: Convergence patterns for Braess network (SB-DSO)

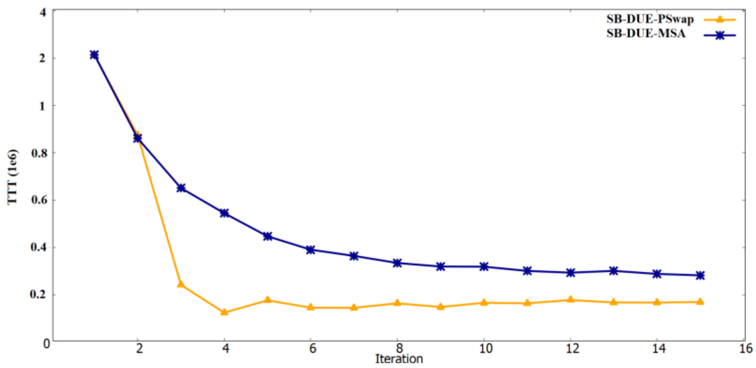


Figure 5: Convergence patterns for Braess network (SB-DUE)

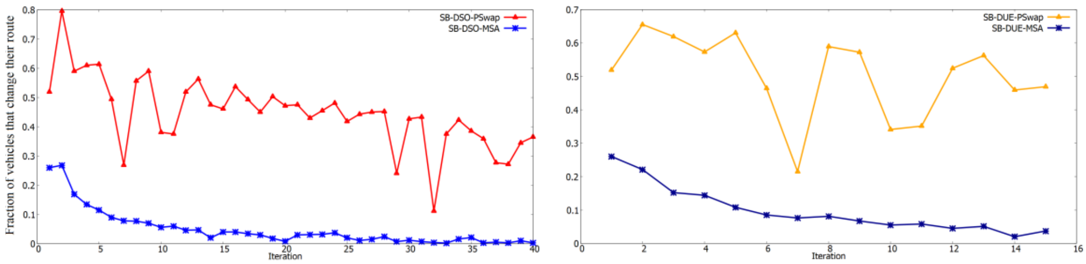


Figure 6: Fraction of vehicles that change their route per iteration (Braess Network)

Figures 4 and 5 show that MSA is dominated by PSwap. The TTT percentage difference (for the best iteration) between PSwap and MSA for the SB-DSO algorithm and SB-DUE algorithm are equal to 49.19% and 55.74%, respectively, which suggests the superior performance of the PSwap. The summary statistics for the system performance under different scenarios (best iterations) are reported in Table 5.

Table 5: Simulation Results for Braess Network

Scenario	TTT: Total Travel Time (s)	AS: Average Speed (m/s)	ADT: Average Distance Travelled (m)	ATL: Average Time Loss (s)
SB-DSO-PSwap	120130	8.69	1002.99	49.30
SB-DSO-MSA	236430	5.40	1007.10	160.82
SB-DUE-PSwap	125030	8.36	1006.01	56.74
SB-DUE-MSA	282490	4.85	1009.28	184.00

Percentage difference between PSwap and MSA (DSO) = 49.19%
Percentage difference between PSwap and MSA (DUE) = 55.74%
Percentage difference between DSO and DUE (PSwap) = 3.91%
Percentage difference between DSO and DUE (MSA) = 16.3%

As expected, for both swapping algorithms, the SB-DSO has lower TTT than SB-DUE. The Percentage TTT saving of SB-DSO over SB-DUE varies from 3.91% (for PSwap) and 16.30% (for MSA). In addition, an analysis of the data in Table 5 suggests that vehicle compliance with DSO routines increases vehicle AS and decreases vehicle ATL.

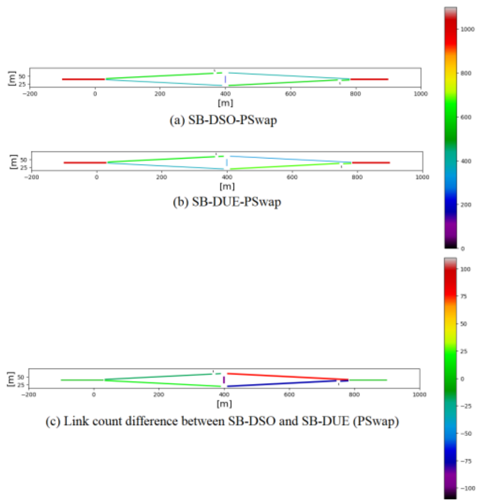


Figure 7: Volume of the Braess network

To examine the superior performance of SB-DSO in more detail, the volume of the Braess network in SB-DSO-PSwap and SB-DUE-PSwap scenarios are illustrated in Figure 7. This figure also shows the traffic volume difference between these two scenarios. Figure 7 (c) demonstrates that when vehicles follow the SB-DSO, they not only use high-speed links (links 1, 5, and 4) to reach their destination, but they also use both high-speed and low-speed links simultaneously. In contrast, in the SB-DUE scenario, most vehicles tend to use high-speed links.

5.2 Random Network

A Random network was generated in SUMO (Figure 2 (b)) using *netgenerate*. This network consists of 278 edges and 100 junctions. Each edge has a minimum length of 200 and a maximum length of 1000 meters. The number of lanes is either one or two for each edge. A random traffic demand of 7200 vehicles was generated for a one-hour simulation. These vehicles were randomly distributed to the network. Similar to the previous test network, four scenarios were evaluated for the Random network: (1) SB-DSO-PSwap, (2) SB-DSO-MSA, (3) SB-DUE-PSwap, and (4) SB-DUE-MSA. The microsimulation is performed for the scenarios, all of which converged after 17 iterations. The convergence pattern of the SB-DSO and SB-DUE algorithms are displayed in Figure 8 and Figure 9, respectively. Besides, the fraction of re-routing vehicles per iteration for each scenario is given in figure 10.

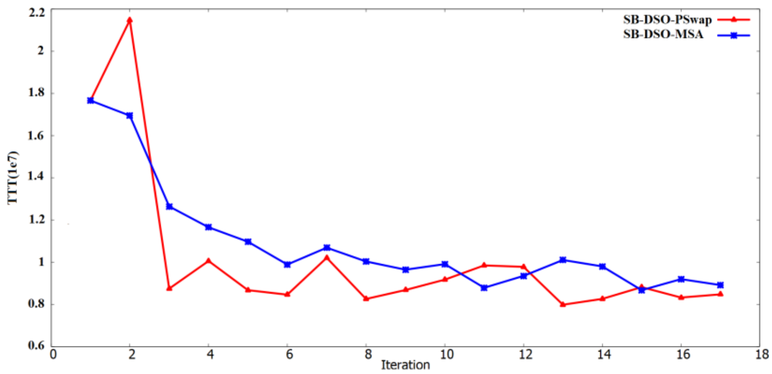


Figure 8: Convergence patterns for Random network (SB-DSO)

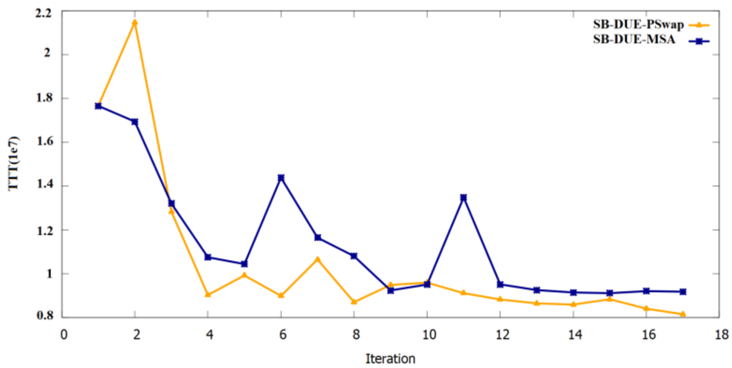


Figure 9: Convergence patterns for Random network (SB-DUE)

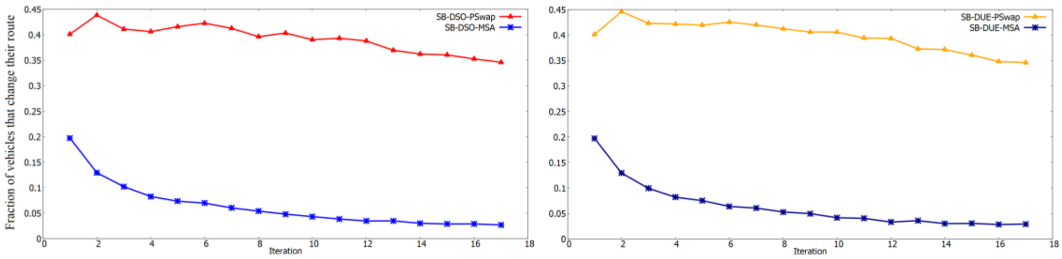


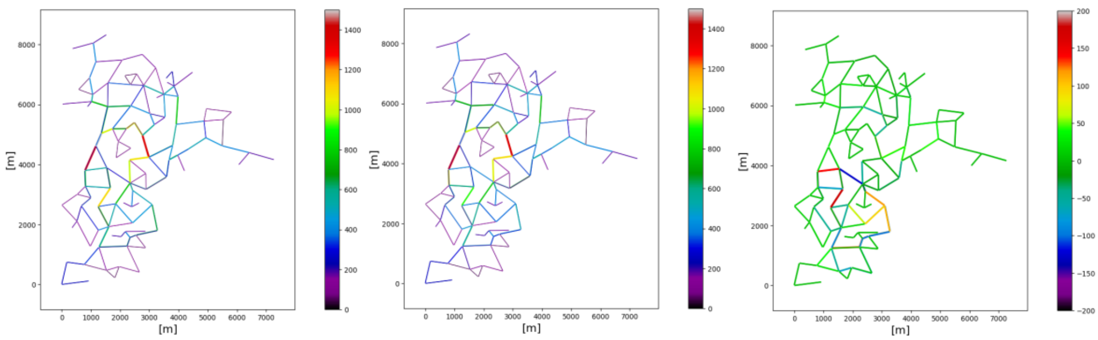
Figure 10: Fraction of vehicles that change their route per iteration (Random Network)

For both SB-DSO and SB-DUE, the value at convergence for PSwap is lower than MSA. Also, PSwap has better performance in terms of stability of the convergence and the TTT average value. The TTT percentage difference between PSwap and MSA for SB-DSO and SB-DUE are equal to 7.95% and 10.53%, respectively, which supports that PSwap has superior acting. The results of the optimum iteration for each scenario are presented in Table 6.

Table 6: Simulation Results for Random Network

Scenario	TTT: Total Travel Time (s)	AS: Average Speed (m/s)	ADT: Average Distance Travelled (m)	ATL: Average Time Loss (s)
SB-DSO-PSwap	7981776	8.77	6850.82	578.09
SB-DSO-MSA	8671176	8.52	6571.42	677.58
SB-DUE-PSwap	8157456	8.70	6787.33	599.06
SB-DUE-MSA	9118296	8.51	6581.33	761.72
Percentage difference between PSwap and MSA (DSO) = 7.95%				
Percentage difference between PSwap and MSA (DUE) = 10.53%				
Percentage difference between DSO and DUE (PSwap) = 2.15%				
Percentage difference between DSO and DUE (MSA) = 4.90%				

As expected, the TTT value decreased by 2.15% (PSwap) and 4.90% (MSA) in the SB-DSO scenario compared to that in the SB-DUE scenario. In addition to the reduction in TTT, we observed a decrease of 3.5% (PSwap) and 11% (MSA) in ATL. However, the ADT by vehicles in the SB-DSO scenario shows a slight increment over the SB-DUE scenario. This suggests that vehicles do not necessarily select routes that have the shortest distance in the SB-DSO scenario but rather select those that reduce the travel time of the entire network (TTT).



(a) SB-DSO-PSwap

(b) SB-DUE-PSwap

(c) Link count difference between SB-DSO and SB-DUE (PSwap)

Figure 11: Volume of the Random network

Figure 11 shows traffic volume in the Random network in different scenarios. The comparison of Figures 11 (a) and Figure 11 (b) reveals that in the SB-DSO scenario, the number of medium-volume links is slightly more than that of the SB-DUE scenario. This indicates that in the SB-DSO scenario, the traffic volume is distributed throughout the entire network. In contrast, in the SB-DUE scenario, only a limited number of short links are used. A similar result can be obtained by observing Figure 8 (c). In Figure 8 (c), the red highlighted links represent the links used by vehicles in the SB-DSO scenario and not in the SB-DUE scenario, while the green highlight indicates the links in which there is no difference in traffic volume between the two scenarios.

5.3 Sioux Falls Network

The latest case study in this article is the Sioux Falls network (Figure 3 (c)). The total number of simulated vehicles was 36000 which were distributed on different origins and destinations based on the demand pattern of LeBlanc's study (LeBlanc et al., 1975). In order to check the performance of the proposed algorithm on the mesoscale, the meso simulation feature of SUMO is implemented for this test network. As in the previous test networks, four scenarios were analyzed. The SB-DSO-PSwap and SB-SO-MSA scenarios converged after 35 iterations (Figure 12), while the SB-DUE-PSwap and SB-DUE-MSA scenarios converged after 25 iterations (Figure 13). The fraction of re-routing vehicles per iteration is also given in figure 14.

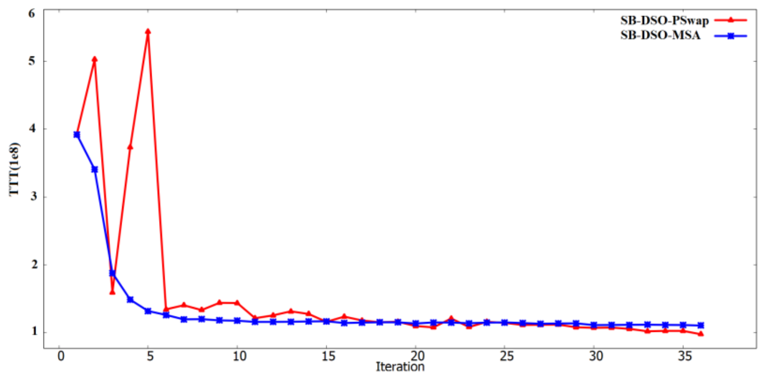


Figure 12: Convergence patterns for Sioux Falls network (SB-DSO)

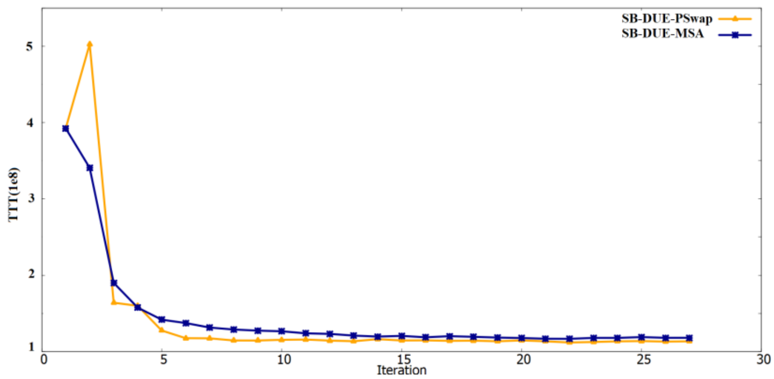


Figure 13: Convergence patterns for Sioux Falls network (SB-DUE)

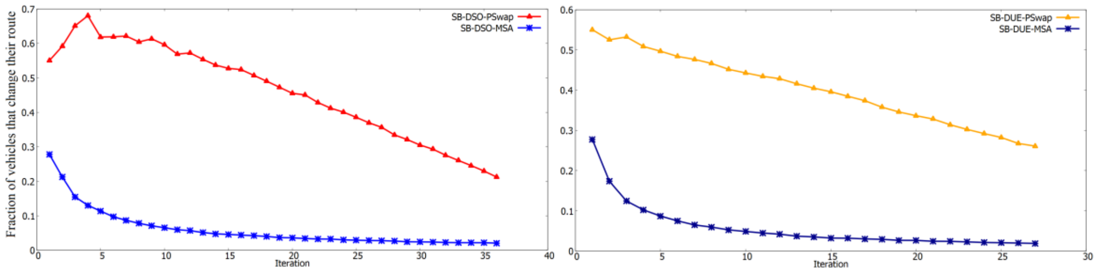


Figure 14: Fraction of vehicles that change their route per iteration (Sioux Falls Network)

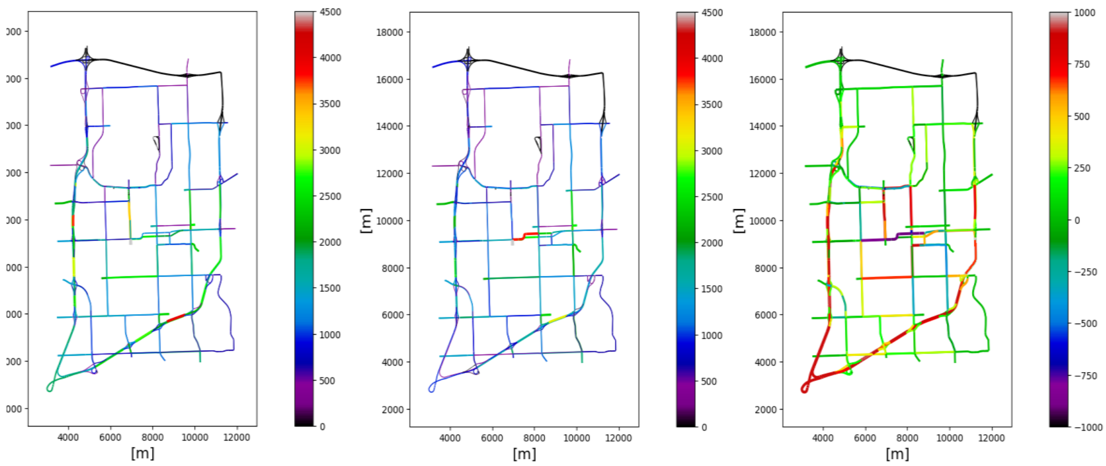
Similar to previous test networks, PSwap has a better value at convergence than MSA. For the SB-DSO-PSwap scenario, the value at convergence is 11.64% lower than that of SB-DSO-MSA. For the SB-DUE algorithm, we can see a similar behavior where the TTT percentage difference between SB-DUE-PSwap and SB-DUE-MSA is 4.1%. The results of the meso simulations are presented in Table 7, which provides traffic-related measures for the Sioux Falls network.

Table 7: Simulation Results for Sioux Falls Network

Scenario	TTT: Total Travel Time (s)	AS: Average Speed (m/s)	ADT: Average Distance Travelled (m)	ATL: Average Time Loss (s)
SB-DSO-PSwap	97488027.9	12.75	9726.57	479.54
SB-DSO-MSA	110334105.4	12.75	9833.79	505.72
SB-DUE-PSwap	111853402.2	11.98	8164.91	526.21
SB-DUE-MSA	116637655.1	11.71	8384.31	595.16

Percentage difference between PSwap and MSA (DSO) = 11.64%
Percentage difference between PSwap and MSA (DUE) = 4.1%
Percentage difference between DSO and DUE (PSwap) = 12.84%
Percentage difference between DSO and DUE (MSA) = 5.4%

The simulation results indicate that the proposed SB-DSO traffic assignment leads to a reduction of 12.84% (PSwap) and 5.4% (MSA) in TTT and 8.86% (PSwap) and 15% (MSA) in ATL. In addition, the AS in SB-DSO was improved compared to that in SB-DUE (6% and 8.15% increase in AS for PSwap and MSA, respectively). However, ADT has increased in the DSO condition (16% and 14.7% increment of ADT for PSwap and MSA, respectively).



(a) SB-DSO-PSwap

(b) SB-DUE-PSwap

(c) Link count difference between SB-DSO and SB-DUE (PSwap)

Figure 15: Volume of the Sioux Falls network

Figure 15 shows the volume on the Sioux Falls network for the proposed SB-DSO-PSwap and the current SB-DUE-PSwap, categorized based on both colors and width, where thicker links indicate higher volume. Focusing on the difference between the two traffic assignment methods, Figure 15 (c) shows that vehicles are distributed among the entire network in SB-DSO scenarios and do not intend to use specific short links that minimize their own travel time (avoiding selfish routing). Instead, they select unused links, minimizing the travel time in the entire network. In other words, the presence of short minor roads may have a significant impact on SB-DSO versus SB-DUE. This is not because the network can handle more load but because shorter roads provide opportunities for selfish shortcuts that are prone to jamming.

6 Conclusion

SO and UE traffic assignments are among the most important traffic assignment methods and have been extensively investigated for several years. Although recent studies suggest that several road users (e.g., CAVs) are soon expected to follow the SO principle, most traffic simulation packages do not provide it. In addition, the majority of available SO solutions for CAVs are either static or analytical which have their own drawbacks. On the other hand, the available simulation-based methods are based on the MSA (or its extensions) algorithm, which has its own drawbacks. Therefore, this study proposes a new SB-DSO traffic assignment algorithm that replaces the travel times of links with a surrogate model of the MTT. The logit route choice model is incorporated in the solution algorithm. At each iteration, the route choice model is applied to the path set of each vehicle. A new swapping algorithm (Called PSwap) is presented, which is based on the logit route choice model to address the disadvantages of MSA (that uses an all-or-nothing assignment). The swapping algorithm prevents all vehicles from changing their routes in successive iterations. The proposed algorithm is tested on two classical case studies (Braess and Sioux Falls network) and a random network to assess its performance (both micro and meso scale). For each test network, four scenarios have been simulated: (1) SB-DSO-PSwap, (2) SB-DSO-MSA, (3) SB-DUE-PSwap, and (4) SB-DUE-MSA. The results of the simulations show that MSA is dominated by PSwap in all of the scenarios. Also, a comparison of the proposed SB-DSO (scenarios 1 and 2) and current SB-DUE (scenarios 3 and 4) traffic assignment algorithm is provided. We observed remarkable decreases in the TTT when vehicles followed the SB-DSO. The maximum percentage of TTT reduction was for the Braess network (16.3%), followed by 12.84% and 4.9% for Sioux Falls and Random networks, respectively. In summary, the proposed SB-DSO-PSwap has the least amount of TTT. These results indicate that if road users (such as CAVs) follow SO routines in the future, a significant reduction in travel time and pollution of the entire network can be obtained, which, accordingly, reduces several costs.

One of the most critical factors in the superiority of SB-DSO over SB-DUE is the presence of short minor roads, as these routes provide opportunities for selfish routing. Therefore, the proposed algorithm may not necessarily improve the TTT in networks where short minor roads (or alternative routes) are not present. Also, it should be pointed out that the number of teleporting vehicles has a considerable impact on the results of the TTT. Therefore, it is not appropriate to evaluate the performance of the algorithm if the number of teleporting vehicles is very high.

The proposed algorithm is freely available under the EPLv2 license on GitHub (Eclipse, 2022) by setting *--marginal-cost*, *--marginal-cost.exp*, and *--convergence-steps* options in *duaIterate.py*. As the MTTs in the proposed algorithm are calculated based on a local approximation, it may lead to its overestimation. Therefore, it is recommended to remove the second term of the MTT equation in case of inappropriate results (by removing *--marginal-cost.exp* option). This tool helps researchers and decision-makers in evaluating the effect of SO-seeking users (e.g., CAVs) on the road network in terms of traffic and environment-related issues. Beyond solving the SB-DTA problem by a new swapping algorithm, proposing a surrogate model for MTT is helpful in many road network management applications, such as providing marginal cost-based tolls.

Future studies should estimate the global approximation of MTTs. In this study, the examined scenarios were scenarios in which all vehicles followed either DSO or DUE rules; therefore, it is suggested that a mixed traffic algorithm should be developed in future research. In addition, owing to the importance of the demand level in traffic assignment, researchers are encouraged to assess the impact of different demand levels on the proposed SB-DSO traffic assignment algorithm. Another direction for future research is the comparison of environmental-related measures in DSO and DUE conditions.

7 Contribution and Disclosure statement

Behzad Bamdad Mehrabani: Conceptualization, Methodology, Software, Investigation, Data Curation, Writing – original draft, Visualization. **Jakob Erdmann**: Methodology, Software, Validation, Formal Analysis, Resources, Data Curation, Writing – review & editing. **Luca Sgambi**:, Validation, Writing – review & editing, Supervision, Project Administration, Funding Acquisition. **Maaike Snelder**: Conceptualization, Methodology, Validation, Writing – review & editing.

No potential conflict of interest was reported by the author(s).

References

Ameli, M., Lebacque, J.-P., & Leclercq, L. (2020a). Improving traffic network performance with road banning strategy: A simulation approach comparing user equilibrium and system optimum. *Simulation Modelling Practice and Theory*, *99*, 101995. https://doi.org/https://doi.org/10.1016/j.simpat.2019.101995

Ameli, M., Lebacque, J., & Leclercq, L. (2020b). Simulation-based dynamic traffic assignment: Meta-heuristic solution methods with parallel computing. *Computer-Aided Civil and Infrastructure Engineering*, *35*(10), 1047–1062.

Bagloee, S. A., Sarvi, M., Patriksson, M., & Rajabifard, A. (2017). A mixed user-equilibrium and system-optimal traffic flow for connected vehicles stated as a complementarity problem. *Computer-Aided Civil and Infrastructure Engineering*, *32*(7), 562–580. https://doi.org/https://doi.org/10.1111/mice.12261

Bamdad Mehrabani, B., Sgambi, L., Garavaglia, E., & Madani, N. (2021). Modeling Methods for the Assessment of the Ecological Impacts of Road Maintenance Sites. In P. Singh (Ed.), *Environmental Sustainability and Economy 1st Edition*. Elsevier. https://www.elsevier.com/books/environmental-sustainability-and-economy/singh/978-0-12-822188-4

Barceló, J, & Ferrer, J. L. (1997). Advanced Interactive Microscopic Simulator for Urban Network. *Departement d'Estadística i Investigació Operativa, Universitat Politèctina de Catalunya, User's Manual Edn*.

Barceló, Jaume. (2010). *Fundamentals of traffic simulation* (Vol. 145). Springer. https://doi.org/10.1007/978-1-4419-6142-6_1

Ben-Akiva, M. E., Koutsopoulos, H. N., Mishalani, R. G., & Yang, Q. (1997). Simulation laboratory for evaluating dynamic traffic management systems. *Journal of Transportation Engineering*, *123*(4), 283–289. https://doi.org/https://doi.org/10.1061/(ASCE)0733-947X(1997)123:4(283)

Chen, R., Becarie, C., & Leclercq, L. (2021). Learning link marginals from dynamic simulation to calculate sustainable system optimum. *HEART 2020, 9th Symposium of the European Association for Research in Transportation-Virtual Conference*, 13p. https://hal.archives-ouvertes.fr/hal-03154849

Chiu, Y. C., Nava, E., Zheng, H., & Bustillos, B. (2011). DynusT user's manual. In *Department of Engineering, University of Arizona, Tucson*.

DLR. (2021). *SUMO User Documentation*. https://sumo.dlr.de/docs/index.html

Doan, K., & Ukkusuri, S. V. (2015). Dynamic system optimal model for multi-OD traffic networks with an advanced spatial queuing model. *Transportation Research Part C: Emerging Technologies*, *51*, 41–65. https://doi.org/https://doi.org/10.1016/j.trc.2014.10.011

Dobler, C., & Nagel, K. (2016). Within-day replanning. In *The Multi-Agent Transport Simulation MATSim* (pp. 187–200). Ubiquity Press. https://doi.org/http:// dx.doi.org/10.5334/baw.30

Eclipse. (2022). *Eclipse SUMO*. https://github.com/eclipse/sumo/blob/master/tools/assign/duaIterate.py

Fellendorf, M. (1996). VISSIM for traffic signal optimisation. *Traffic Technology International'96. Annual Review Issue*. http://worldcat.org/issn/13569252

Gawron, C. (1998). *Simulation-Based Traffic Assignment. Computing user equilibria in large street networks* [Universität zu Köln]. http://kups.ub.uni-koeln.de/id/eprint/9257

Ghali, M. O., & Smith, M. J. (1995). A model for the dynamic system optimum traffic assignment problem. *Transportation Research Part B: Methodological*, *29*(3), 155–170. https://doi.org/https://doi.org/10.1016/0191-2615(94)00024-T

Gipps, P. G. (1981). A behavioural car-following model for computer simulation. *Transportation Research Part B: Methodological*, *15*(2), 105–111. https://doi.org/https://doi.org/10.1016/0191-2615(81)90037-0

Hu, T.-Y., Tong, C.-C., Liao, T.-Y., & Chen, L.-W. (2018). Dynamic route choice behaviour and simulation-based dynamic traffic assignment model for mixed traffic flows. *KSCE Journal of Civil Engineering*, *22*(2), 813–822. https://doi.org/10.1007/s12205-017-1025-8

Jayakrishnan, R., Mahmassani, H. S., & Hu, T.-Y. (1994). An evaluation tool for advanced traffic information and management systems in urban networks. *Transportation Research Part C: Emerging Technologies*, *2*(3), 129–147. https://doi.org/https://doi.org/10.1016/0968-090X(94)90005-1

Krauß, S., Wagner, P., & Gawron, C. (1997). Metastable states in a microscopic model of traffic flow. *Physical Review E*, *55*(5), 5597. https://doi.org/https://doi.org/10.1103/PhysRevE.55.5597

Lämmel, G., & Flötteröd, G. (2009). Towards system optimum: Finding optimal routing strategies in time-dependent networks for large-scale evacuation problems. *Annual Conference on Artificial Intelligence*, 532–539. https://doi.org/https://doi.org/10.1007/978-3-642-04617-9_67

LeBlanc, L. J., Morlok, E. K., & Pierskalla, W. P. (1975). An efficient approach to solving the road network equilibrium traffic assignment problem. *Transportation Research*, *9*(5), 309–318.

Li, C. (2016). *Multi-agent dynamic traffic assignment incorporating GPS diary and personalized supernetwork approach* [Eindhoven University of Technology]. https://research.tue.nl/en/studentTheses/multi-agent-dynamic-traffic-assignment-incorporating-gps-diary-an

Lighthill, M. J., & Whitham, G. B. (1955). On kinematic waves II. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, *229*(1178), 317–345. https://doi.org/https://doi.org/10.1098/rspa.1955.0089

Liu, H. X., He, X., & He, B. (2009). Method of successive weighted averages (MSWA) and self-regulated averaging schemes for solving stochastic user equilibrium problem. *Networks and Spatial Economics*, *9*(4), 485–503.

Liu, J., Mirchandani, P., & Zhou, X. (2020). Integrated vehicle assignment and routing for system-optimal shared mobility planning with endogenous road congestion. *Transportation Research Part C: Emerging Technologies*, *117*, 102675. https://doi.org/https://doi.org/10.1016/j.trc.2020.102675

Liu, R. (2010). Traffic simulation with DRACULA. In *Fundamentals of traffic simulation* (pp. 295–322). Springer. https://doi.org/https://doi.org/10.1007/978-1-4419-6142-6_8

Lopez, P. A., Behrisch, M., Bieker-Walz, L., Erdmann, J., Flötteröd, Y.-P., Hilbrich, R., Lücken, L., Rummel, J., Wagner, P., & Wießner, E. (2018). Microscopic traffic simulation using sumo. *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, 2575–2582. https://doi.org/10.1109/ITSC.2018.8569938

Lu, C.-C., Liu, J., Qu, Y., Peeta, S., Rouphail, N. M., & Zhou, X. (2016). Eco-system optimal time-dependent flow assignment in a congested network. *Transportation Research Part B: Methodological*, *94*, 217–239. https://doi.org/https://doi.org/10.1016/j.trb.2016.09.015

Mahmassani, H. S., & Peeta, S. (1993). Network performance under system optimal and user equilibrium dynamic assignments: implications for advanced traveler information systems. *Transportation Research Record*, *1408*, 83. http://onlinepubs.trb.org/Onlinepubs/trr/1993/1408/1408-011.pdf

Mahmassani, H. S., & Peeta, S. (1995). System optimal dynamic assignment for electronic route guidance in a congested traffic network. In *Urban Traffic Networks* (pp. 3–37). Springer. https://doi.org/https://doi.org/10.1007/978-3-642-79641-8_1

Mahut, M. (2001). *A DISCRETE FLOW MODEL FOR DYNAMIC NETWORK LOADING* [Universitk de Montreal]. https://trid.trb.org/view/660668

Mansourianfar, M. H., Gu, Z., Waller, S. T., & Saberi, M. (2021). Joint routing and pricing control in congested mixed autonomy networks. *Transportation Research Part C: Emerging Technologies*, *131*, 103338.

Newell, G. F. (2002). A simplified car-following theory: a lower order model. *Transportation Research Part B: Methodological*, *36*(3), 195–205. https://doi.org/https://doi.org/10.1016/S0191-2615(00)00044-8

Ngoduy, D., Hoang, N. H., Vu, H. L., & Watling, D. (2021). Multiclass dynamic system optimum solution for mixed traffic of human-driven and automated vehicles considering physical queues. *Transportation Research Part B: Methodological*, *145*, 56–79. https://doi.org/https://doi.org/10.1016/j.trb.2020.12.008

Patriksson, M. (2015). *The traffic assignment problem: models and methods*. Courier Dover Publications.

Peeta, S., & Mahmassani, H. S. (1995). System optimal and user equilibrium time-dependent traffic assignment in congested networks. *Annals of Operations Research*, *60*(1), 81–113. https://doi.org/https://doi.org/10.1007/BF02031941

Peeta, S., & Ziliaskopoulos, A. K. (2001). Foundations of dynamic traffic assignment: The past, the present and the future. *Networks and Spatial Economics*, *1*(3), 233–265. https://doi.org/https://doi.org/10.1023/A:1012827724856

Qian, Z. S., Shen, W., & Zhang, H. M. (2012). System-optimal dynamic traffic assignment with and without queue spillback: Its path-based formulation and solution via approximate path marginal cost. *Transportation Research Part B: Methodological*, *46*(7), 874–893. https://doi.org/https://doi.org/10.1016/j.trb.2012.02.008

Rahman, M. R., Siddique, A., & Shahid Mamun, M. (2015). Comparison of User Equilibrium (UE) and System Optimum (SO) Traffic Assignment Methods for Auto Trips. *International Conference on Recent Innovation in Civil Engineering for Sustainable Development (IICSD-2015*. https://doi.org/10.13140/RG.2.1.2169.3204

Richards, P. I. (1956). Shock waves on the highway. *Operations Research*, *4*(1), 42–51. https://doi.org/https://doi.org/10.1287/opre.4.1.42

Saw, K., Katti, B. K., & Joshi, G. (2015). Literature review of traffic assignment: static and dynamic.

International Journal of Transportation Engineering, *2*(4), 339–347. https://doi.org/10.22119/IJTE.2015.10447

Sbayti, H., Lu, C.-C., & Mahmassani, H. S. (2007). Efficient implementation of method of successive averages in simulation-based dynamic traffic assignment models for large-scale network applications. *Transportation Research Record*, *2029*(1), 22–30. https://doi.org/https://doi.org/10.3141/2029-03

Sheffi, Y. (1985). *Urban transportation networks* (Vol. 6). Prentice-Hall, Englewood Cliffs, NJ.

Shen, W., Nie, Y., & Zhang, H. M. (2007). On path marginal cost analysis and its relation to dynamic system-optimal traffic assignment. *Transportation and Traffic Theory 2007. Papers Selected for Presentation at ISTTT17Engineering and Physical Sciences Research Council (Great Britain) Rees Jeffreys Road FundTransport Research FoundationTMS ConsultancyOve Arup and Partners, Hong KongTransp*. http://worldcat.org/isbn/9780080453750

Shen, W., Nie, Y., & Zhang, H. M. (2006). Path-based system optimal dynamic traffic assignment models: formulations and solution methods. *2006 IEEE Intelligent Transportation Systems Conference*, 1298–1303. https://doi.org/10.1109/ITSC.2006.1707402

Shen, W., & Zhang, H. M. (2009). On the morning commute problem in a corridor network with multiple bottlenecks: Its system-optimal traffic flow patterns and the realizing tolling scheme. *Transportation Research Part B: Methodological*, *43*(3), 267–284. https://doi.org/https://doi.org/10.1016/j.trb.2008.07.004

Smith, M., Duncan, G., & Druitt, S. (1995). PARAMICS: microscopic traffic simulation for congestion management. *IEE Colloquium on Dynamic Control of Strategic Inter-Urban Road Networks*, 1–8. https://doi.org/10.1049/ic:19950249

Taale, H., & Pel, A. (2015). Better convergence for dynamic traffic assignment methods. *Transportation Research Procedia*, *10*, 197–206.

Tajtehranifard, H., Bhaskar, A., Nassir, N., Haque, M. M., & Chung, E. (2018). A path marginal cost approximation algorithm for system optimal quasi-dynamic traffic assignment. *Transportation Research Part C: Emerging Technologies*, *88*, 91–106. https://doi.org/https://doi.org/10.1016/j.trc.2018.01.002

Taylor, N. B. (2003). The CONTRAM dynamic traffic assignment model. *Networks and Spatial Economics*, *3*(3), 297–322. https://doi.org/https://doi.org/10.1023/A:1025394201651

Tsanakas, N. (2019). *Emission estimation based on traffic models and measurements* (Vol. 1835) [Linköping University Electronic Press]. https://doi.org/10.3384/lic.diva-155771

US-DOT. (1995). *TRAF User Reference Guide*. Federal Highway Administration.

Van Aerde, M., Hellinga, B., Baker, M., & Rakha, H. (1996). INTEGRATION: An overview of traffic simulation features. *Transportation Research Records*. http://www.civil.uwaterloo.ca/bhellinga/publications/Publications/TRB 1996 Integration Features.pdf

Wang, J., Peeta, S., & He, X. (2019). Multiclass traffic assignment model for mixed traffic flow of human-driven vehicles and connected and autonomous vehicles. *Transportation Research Part B: Methodological*, *126*, 139–168. https://doi.org/https://doi.org/10.1016/j.trb.2019.05.022

Wang, W., Uehara, M., & Ozaki, H. (2018). System optimum traffic assignment for connected cars. *2018 Sixth International Symposium on Computing and Networking Workshops (CANDARW)*, 370–375. https://doi.org/10.1109/CANDARW.2018.00074

Wardrop, J. G. (1952). Road paper. some theoretical aspects of road traffic research. *Proceedings of the Institution of Civil Engineers*, *1*(3), 325–362. https://doi.org/https://doi.org/10.1680/ipeds.1952.11259

Wie, B.-W., Friesz, T. L., & Tobin, R. L. (1990). Dynamic user optimal traffic assignment on congested multidestination networks. *Transportation Research Part B: Methodological*, *24*(6), 431–442. https://doi.org/https://doi.org/10.1016/0191-2615(90)90038-Z

Yang, I., & Jayakrishnan, R. (2012). Gradient Projection Method for Simulation-Based Dynamic

Traffic Assignment. *Transportation Research Record*, *2284*(1), 70–80. https://doi.org/10.3141/2284-09

Yperman, I. (2007). *The link transmission model for dynamic network loading* [KU LEUVEN].

Zhang, P., & Qian, S. (2020). Path-based system optimal dynamic traffic assignment: A subgradient approach. *Transportation Research Part B: Methodological*, *134*, 41–63. https://doi.org/https://doi.org/10.1016/j.trb.2020.02.004

Zhou, X., & Taylor, J. (2014). DTALite: A queue-based mesoscopic traffic simulator for fast model evaluation and calibration. *Cogent Engineering*, *1*(1). https://doi.org/https://doi.org/10.1080/23311916.2014.961345

Ziliaskopoulos, A. K. (2000). A linear programming model for the single destination system optimum dynamic traffic assignment problem. *Transportation Science*, *34*(1), 37–49.