

Bus Priority Procedure for Signalized Intersections Based on Bus Occupancy and Delay

Katia Juliane Schmidt^{1*}, Natalie Steinmetz¹, and Martin Margreiter¹

¹Technical University of Munich, Germany

*Correspondence: Katia Juliane Schmidt, katia.schmidt@tum.de

Abstract. This study proposes a bus prioritisation strategy at signalised intersections to enhance public transport reliability and attractiveness. Nowadays, bus prioritisation at intersections is conducted according to a first-come, first-served principle, lacking compatibility with future vehicle-to-infrastructure communication. A framework for prioritising buses based on their delay and occupancy was developed and tested in a SUMO microscopic traffic simulation subnetwork of the city of Ingolstadt. Buses are prioritised using a 25-level hierarchy. Four degrees of prioritisation interventions are implemented based on bus priority levels with signal cycles adjusted to advance preferred green phases. The timing of the prioritisation is based on an Estimated Time of Arrival (ETA) prediction that considers past speed and travel time data as well as bus stops that are on the way to the intersection. The prioritisation logic was tested in simulation scenarios with on- and off-peak conditions and with several buses requesting priority and varying degrees of priority. The results show that the developed prioritisation concept works, and prioritised buses benefit from a strong reduction in their travel times (up to 87 %) and number of stops. Buses with lower priority levels may experience deterioration in their travel time (up to 126 %) when arriving at the same time as a high-priority bus, but considering the fewer affected passengers and smaller delay, this seems acceptable.

Keywords: Adaptive Bus Prioritisation, Traffic Control, Acceleration of Public Transport

1. Introduction

The fundamental concept behind bus prioritisation is to optimise the speed and efficiency with which the bus moves through the network. Various strategies can be employed to realise this objective, encompassing the establishment of dedicated bus lanes, enhancements to bus stops, and adjustments to traffic signal programs. The imperative for prioritisation arises from challenges related to the punctuality and reliability of public transport services, particularly in scenarios involving mixed traffic conditions and unfavourable signalisations. Ingolstadt, a city in Upper Bavaria and distinguished as the site of one of the largest automobile manufacturing plants in Germany, has articulated several related objectives and initiatives within its Traffic Development Plan 2025 [1]. The city aims to shift the modal split towards public transport while optimising traffic flow and safety, particularly at intersections. The research initiative of recent years known as KIVI, an acronym for "Artificial Intelligence in the Transportation System of Ingolstadt," has made a significant contribution to these goals [2]. The city of Ingolstadt together with various economic and scientific partners took part in the KIVI project, which aimed to increase traffic safety, optimise traffic flow and enhance the performance of existing traffic infrastructure [3],[4].

This paper proposes a further development of bus prioritisation for rule-based and traffic-actuated signal control, which is the predominant form of traffic signal control in the DACH

region [2]. With this proposed enhancement of bus prioritisation, vehicles are prioritised based on their levels of delay and occupancy – a paradigm shift away from the prevalent first-come, first-served principle, providing a more dynamic and responsive approach. The envisioned future scenarios are tested in SUMO simulations at the intersection of Goethestrasse/Schillerstrasse in the city centre of Ingolstadt, a crucial component of the high-definition test field established within the KIVI project. By intricately examining the interplay between intervention strategies and prioritisation methodologies, this study seeks to offer comprehensive insights into the potential improvements in bus services within a simulated urban environment.

This paper covers established prioritisation strategies in section 2, followed by section 3, which outlines the methodology underpinning this study. Section 4 encompasses the simulation results, in section 5 discussions on influencing factors and mitigation strategies follow and section 6 concludes with recommendations for future research.

2. Literature Review

2.1 Introduction to Public Transport Prioritisation

Prioritisation strategies for public transport can be realised with different approaches such as organisational measures, construction, and technological measures. Organisational schemes can reduce conflict points through new line alignments, a change in the right-of-way at intersections, moving taxi hotspots and others [5]. Separating modes and accelerating the boarding and alighting procedures through bus lanes, improvements in vehicle design or the relocation of stop facilities all fall into the realm of construction measures. The technological measure, public transport signal priority, may be described as “an operational strategy that facilitates the movement of transit vehicles (usually those in-service), either buses or streetcars, through traffic-signal controlled intersections” [6]. Public transport signal priority can be implemented through passive strategies, where public transport vehicles are not specifically detected, and through active prioritisation strategies, which utilises detection of public transport vehicles to adapt the signal timing favourably for the vehicles [7], [8].

2.2 Passive Prioritisation

Lin et al. [7] describe four passive prioritisation strategies: adjusting cycle lengths and signal timings, splitting phases, applying an area wide signal timing plan and metering the overall vehicle flow. According to Machemehl [9] long cycle lengths increase the vehicle throughput of an intersection. However, if phases are very long, the potential wait times and thus the delay for public transport vehicles also increases, therefore, shorter cycle lengths may be preferential. Benefits according to NACTO [10] include reduced waiting times for pedestrians and Furth and SanClemente [11] additionally describe that shorter cycles result in shorter queues for more frequently served approaches. Another approach to reduce the waiting times at the stop line is to split the green time for buses into several shorter phases, which are then reserved throughout one cycle, which was studied by Garrow and Machemehl in 1998 [12]. Realising an area wide signal plan, Lin et al. [7] describe the implementation of a green wave according to bus speed instead of car speed. Therefore, the dwell time at bus stops and the time for acceleration and deceleration of the bus is considered. The benefits of slower green waves include safer urban streets with slower vehicles, which is also beneficial for cyclists [13]. Metering vehicles as a passive prioritisation strategy is described by Urbanik and Holder [14]. The idea is to restrict the number of cars entering the intersection or approach to ensure a reliable bus service. However, according to Lin et al. [7] it is rarely used in practice because of the high impact on traffic flow.

2.3 Active Prioritisation

The Transit Street Design Guide defines active prioritisation as a tool that adapts the traffic signal timing or the phasing if public transport vehicles are detected [8]. The Federal Transit Administration also emphasises the detection of the public transportation vehicle, ensuring that prioritisation only occurs when a public transport vehicle is present [15]. Based on this detection and possibly further conditions, such as delay, the vehicle is given prioritisation [15]. Prioritisation strategies can be differentiated between rule-based and model-based concepts.

In rule-based strategies, prioritisation is determined based on a predefined set of rules. Ma and Bai [16] and He et al. [17] both studied multiple requests for priority. Ma and Bai [16] developed a decision tree for both methods with the aim of minimising the average person delay. He et al. [17] developed an algorithm to deal with multiple priority requests optimising signal timing at an isolated intersection considering all priority requests. They showed that this method was able to reduce bus delay by up to 50 % and was especially effective in dealing with conflicting priority requests. Various studies looked at strategies to reduce the impact that public transport signal priority can have on non-prioritised road users. Allsop [18] suggested to only give priority if no priority treatment was executed in the previous cycle. Similarly, Evans and Skiles [19] proposed to only give priority if red truncation was not applied in the previous cycle. Another approach to decrease the impacts of prioritisation treatments is to apply compensation by extra green time for non-prioritised approaches. This was studied by Gallivan et al. [20], as well as El-Reedy and Ashworth [21], Cooper et al [22] and Collier [23]. It was shown that compensation measures cause additional delay. Bowen et al. [24] studied the integration of transit signal control in SCOOTs (split cycle offset optimisation technique). Hounsell et al. [25] found that green extension as public transport signal priority treatment has the best overall impact while red truncation imposes a high cost on non-public transport vehicles. Liao and Davis [26] developed an adaptive conditional priority control in Minnesota using schedule adherence, number of passengers, dwelling time and speed of the public transport vehicle as decision criteria.

The model-based strategies consider traffic conditions and bus readiness to apply prioritisation treatments. Head et al. [27] developed a model-based traffic signal control that dealt with multiple priority requests based on the North American dual ring with eight phases. The findings of the model showed a reduction in delay compared to a first-come -first-served strategy. In order to minimise the total person delay, considering both bus passengers and car passengers, various detectors are required to detect both buses and all other vehicles. Chang et al. [28] developed a multi-objective control based on inductive loop data, considering bus passenger delay, car passenger delay and bus schedule adherence in a weighted summation in the objective function. Chang et al. showed that the proposed traffic signal control can reduce the total person delay on the intersection. Lin, Yang, Chang, et al. [29] present a model that did not increase the total person delay while considering the bus passengers' waiting time at the downstream stop. The output of the model is the duration of green extension or red truncation.

2.4 Current and Future Technologies Used in Public Transport Prioritisation

For public transport signal priority to successfully assist vehicles in clearing the intersection, a public transport vehicle must be able to be detected or report its coming arrival at the intersection. In addition, the traffic signal must be able to process the priority request and take actions for prioritisation, if desired [30]. This requires a combination of on-board and wayside technology, including vehicle positioning, communication system between the vehicle and the traffic signal or other roadside infrastructure, signal controllers that are capable of receiving information as well as a traffic management system [8].

Nowadays, public transport prioritisation at signalized intersections works according to the detection point principle at which public transport vehicles send messages at specific locations, usually a pre-request, main-request, and check-out. Gay et al. [30] list various requirements for public transport prioritisation such as an accurate enough prediction of the vehicle's arrival time, and rapid data transmissions [30]. Different vehicle positioning technologies are available, however, in many cases more than one technology is used to ensure accurate positioning of the vehicle: Odometers in combination with beacons or poles and global navigation satellite systems [30]. In most German cities, data transmission systems rely on analogue radio systems to facilitate communication between vehicles and traffic signals, as revealed by a survey conducted by the German Federal Highway Research Institute [30]. This holds true for cities such as Munich and Ingolstadt. While a few cities employ alternative methods like digital radio, Tetra trunked radio, mobile phone networks, or infrared, the predominant approach remains analogue radio. The authors underscore a significant limitation in current systems—the inability to continuously track public transport vehicles. This results from the detection point principle, where only a few messages are exchanged at specific points [30].

Vehicle-to-X (V2X) communication enables vehicles to communicate with other vehicles or other road users as well as the infrastructure [31]. This technology can be used for public transport prioritisation at traffic signals when public transport vehicles want to inform the infrastructure about their arrival. Public transport prioritisation by V2X communication has the benefit that in case of congestion the prioritisation measures can be adapted flexibly. V2X does enable an accurate prognosis of arrival at the stops, which is useful for both the public transport operator as well as for the information of waiting passengers [30]. Gay et al. [30] developed a guideline to convert current technologies for public transport prioritisation into V2X-communication-based prioritisation. The first phase of the transition focuses on the implementation of hardware regarding vehicles and roadside infrastructure, while in the second phase the software is extended. There are various cities which carried out research on V2X-communication-based prioritisation in pilot projects, including but not limited to Braunschweig/Magdeburg with the project SIRENE, Duesseldorf with KoMoD and Kassel with VERONIKA [30]. The prioritisation concept C-Call, central call, by GEVAS utilises both the positioning and speed of the vehicle as well as historical journey data, improving the accuracy for travel time prediction [32].

Passenger occupancy can be determined using various methods at different locations such as on the vehicle or at the stations, with motion or weight sensors as well as through analysis of camera feeds. In addition, mobile data and the records of the ticket sales can be used to ascertain how many passengers are using the network at which time [33]. The messages sent between the public transport vehicles and the infrastructure in an V2X environment are specified in the technical specification ETSI TS 103 301 V2.1.1. (2021-03) [34]. The SREM, signal request extended message, and SSEM, signal request status extended message, sent out by the vehicle and infrastructure contain information about the priority status for the requesting vehicle [34]. This study assumes that in a near-future scenario with a prioritisation procedure utilising V2X communication, information regarding the occupancy and delay of a public transport vehicle will be available to the ITCS system and can be transferred via the V2X messages.

3. Methodology

The essential elements, including input data and underlying assumptions are explored further in this section. It provides a detailed exposition of the prioritisation framework and its guiding principles in section 3.2. The final section 3.3, delves into a comprehensive exploration of the various simulated scenarios, shedding light on the critical parameters employed for evaluation.

3.1 Input Data and Assumptions

The developed bus prioritisation framework is tested in a SUMO simulation of the intersection Goethestrasse/Schillerstrasse which is part of the KIVI research area [35]. In order to model the surrounding traffic realistically, a subnetwork with a 700 metre radius around the intersection was cut out. The complete SUMO simulation network of Ingolstadt is available on GitHub [36]. The following figure 1 shows the subnetwork and highlights the intersection for prioritisation.



Figure 1. Overview of the SUMO subnetwork and intersection used for the simulations.

In the course of cutting out the subnetwork, the vehicle and bus routes, the bus stops and the signal program files were adapted accordingly along with the network file. The routes were calibrated for an average weekday and in a two-stage calibration procedure based on OD matrices from statistical data and traffic counts from induction loop data [37]. The traffic signal program is fixed-time, but hourly adapted based on historical signal timing data that reflects the real traffic-actuated control, see [37] for more details.

The simulated prioritisation in this study envisions a near-future scenario where bus prioritisation is based on V2X communication, enabling a continuous monitoring of buses throughout the network. It assumes the ability to collect continuous information on buses in the network, including distances to the chosen intersection, estimated time of arrival (ETA), line details, route information, and current delay and occupancy. Other vehicles do not necessarily have to be connected, i.e. use V2X communication. The prioritisation scheme is set up in such a way that it is compatible with rule-based traffic-actuated control, as is prevalent in Ingolstadt and elsewhere in the DACH region. The level of intervention in the control system depends on the traffic situation, as is already the case today, but with the additional consideration of delay and occupancy. While the simulation does model various driver types of motorised vehicles, including trucks and delivery vehicles as well as regular cars, and buses, the simulation does not include pedestrians or bicycles. The prioritisation scheme is only applied to one intersection in the subnetwork of Ingolstadt and in this way the effect is spatially limited. Timewise the scenarios are taking place during peak-hour traffic on a weekday between 7:00 and 8:00 and in off-peak traffic during 10:00 and 11:00.

3.2 Prioritisation Logic and Code Framework

Within this work, prioritisation always refers to the traffic signal prioritisation of public transport buses at signalized intersections. The prioritisation should enable different levels of intervention in traffic signal control, which is why different priority degrees are used, resulting from different levels for delay and occupancy. The levels for delay and occupancy are defined separately according to their respective frequencies in the real data from the city of Ingolstadt—delay and occupancy are not weighted in relation to each other due to the unequal range areas. However, because the delay level is assigned first, this is more significant. The levels are also transferrable to other cities, however, the thresholds for the delay and occupancy values can also be changed if desired by the city authorities or the bus service provider. The structures of the levels were formulated by considering delay and occupancy data from all bus lines collected over a four-week period in March 2023. The number of bins has been chosen thoughtfully to enhance clarity in distinguishing interventions for priority. However, special cases are considered, such as delays exceeding 420 seconds or seven minutes. The levels were defined as follows: There are five delay levels ranging from E as the lowest delay to A as the highest delay, and five occupancy levels ranging from five representing the lowest occupancy to one for the highest occupancy level. Their combination results in 25 priority levels, which are assigned to four priority degrees, each priority degree having a target time for the green phase at the requested approach to start; Table 1, showcases this.

Table 1. Priority levels and degrees.

Delay in seconds	Occupancy in persons					Priority Degree	Green Time at
	<= 5: 5	<= 14: 4	<= 30: 3	<= 45: 2	> 46: 1		
<= 59: E	E5	E4	E3	E2	E1	-	-
<= 119: D	D5	D4	D3	D2	D1	D	ETA
<= 239: C	C5	C4	C3	C2	C1	C	ETA-10 sec
<= 420: B	B5	B4	B3	B2	B1	B	ETA-20 sec
> 420: A	A5	A4	A3	A2	A1	A	Check-In

The SUMO simulation is accessed and adapted through the TraCI interface. Every simulation second, various parameters are collected, calculated and stored for each unique bus approaching the intersection, including the line number and distance as well as the ETA, the occupancy and delay value and the resulting priority level. From all buses which are part of the simulation and approaching the intersection, the bus with the highest priority level is identified as “prio bus” and if applicable compared to the bus which assigned priority in the last timestep in case there was one. The comparison generates the to be prioritised bus “prio bus”. This means that a bus is prioritised according to its priority level and not according to the order of arrival as with the prevailing first-come, first-served principle.

Each priority level has a desired start for the green time for the approach the bus takes to the intersection, as listed in Table 1. The maximum green time varies depending on the prioritisation level. For levels C and D, the regular phase length is extended by 30 seconds. Level B, receiving an additional 30 seconds, is extended to a green time lasting a minimum of 50 seconds. At priority level A, buses receive the longest green time, lasting until the estimated moment the bus completes its intersection crossing.

In general, the treatments *red truncation*, *red truncation expanded*, and *green extensions* are possible. Which treatment is selected depends on how close the green time is to the target defined by the priority level. Red Truncation expanded encompasses putting all green phases to a minimum time and the prioritised phase to a maximum. In this way the minimum green times as well as all the intergreen times are kept [38]. Even though the cycle time changes, the order of phases stays the same, and all approaches are serviced. Long red phases are not required here, in comparison to phase switching, where to safely transition from

one phase to another the intergreen times may be longer. The effectiveness and closeness to the intended target time of various treatments depend significantly on when the treatment initiates and at which point in the signal program the modification begins. Therefore, the control logic systematically checks the future traffic signal conditions, starting 42 seconds before the target time and ending two seconds before the target time, at intervals of every two seconds, resulting in 21 options for starting a traffic signal treatment. These 21 available options are evaluated, and the timing and treatment are selected based on the smallest positive difference from the target time.

The ETA value is calculated using the current bus route as well as speed and travel time data obtained from previous simulations. This allows the ETA times in the priority logic to be adjusted for specific intersections. The subpart of the route from the current location to the intersection is checked for planned bus stops. In case no bus stops are to be served until the intersection is reached, the ETA calculation uses the current distance from the stop line and speed data of the buses for the given approach obtained from previous simulations. If bus stops are still to be served, the dwelling time at the last bus stop before the intersection is also considered.

Priority level A is executed when having one or no stops remaining until the intersection or a distance to the intersection of less than 700 meters. Should more stops be enroute or the distance be larger before reaching the intersection, the vehicle will be prioritised with a prioritisation level B, but with level A remaining assigned. For all other prioritisation levels, namely B, C, or D, the system verifies the prioritisation treatment and its start time for accuracy, utilising updated ETA and distance values, as long as the ETA remains greater than the maximum switch time. This value is selected to guarantee that even with an ETA-20 sec green time target, the transition time from any phase to the intended phase remains within the longest switch duration. Furthermore, it can be adjusted as needed to the timings of the phases at other intersections if the logic is applied there.

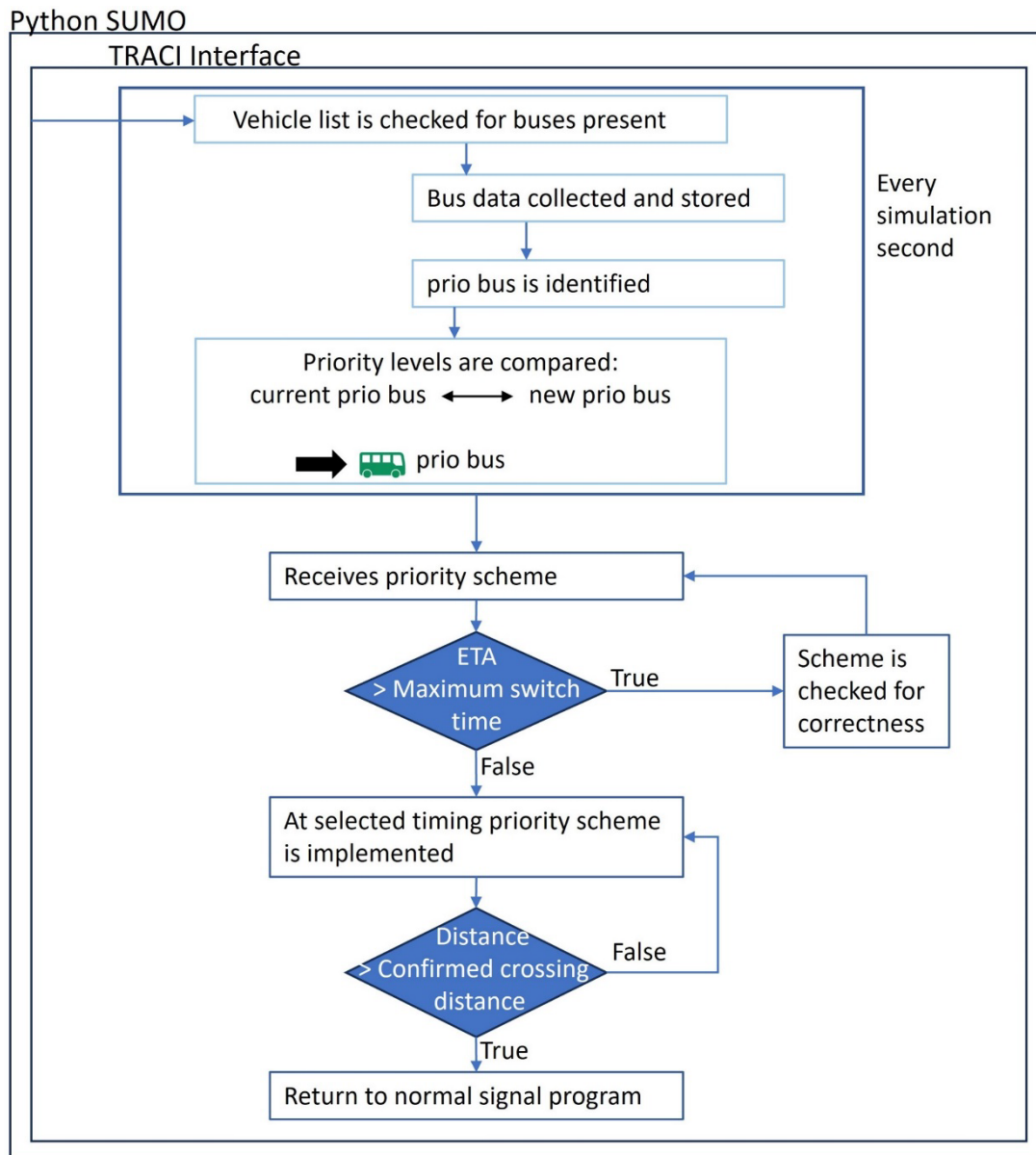


Figure 2. The framework of the prioritisation code.

At the selected timing, the scheme is implemented and the intersection leg the bus is approaching on receives green. At this point in the prioritisation, the distance of the bus is continuously monitored, and once the bus has crossed the stop line and the confirmed crossing distance has been reached, the traffic signal returns to the normal signal operation. The confirmed crossing distance refers to the negative distance the bus travels from the stop line, ensuring that the vehicle has effectively crossed the intersection. Following the confirmed crossing of the public transport vehicle, the opposing intersection approach is given green, in accordance with the minimum green time for the prioritised approach, to reduce queues efficiently. If a higher-prioritised bus is identified during the implementation of or before the initiation of a prioritised bus, the new higher-prioritised bus takes precedence. The prioritisation procedure is terminated, and the system switches to accommodate the new prioritised bus. However, this rule does not apply if the already prioritised bus holds level A prioritisation; in such cases, the prioritisation remains with the original level A bus. This is because the prioritised bus is inherently significant enough to warrant prioritisation, and, in addition, the occurrences of receiving A-level priority, as found in the datasets, are relatively limited.

3.3 Evaluation Scenarios and Parameters

There is an extremely large number of possible scenarios under which the developed prioritisation logic can be tested. In this paper four different scenarios are presented and are compared with scenarios at which buses are not prioritised. In each simulation, two buses traverse the intersection from different approaches. The following figure 3 gives an overview of the bus routes for each scenario with bus P being the prioritised vehicle and bus N not receiving priority treatment.

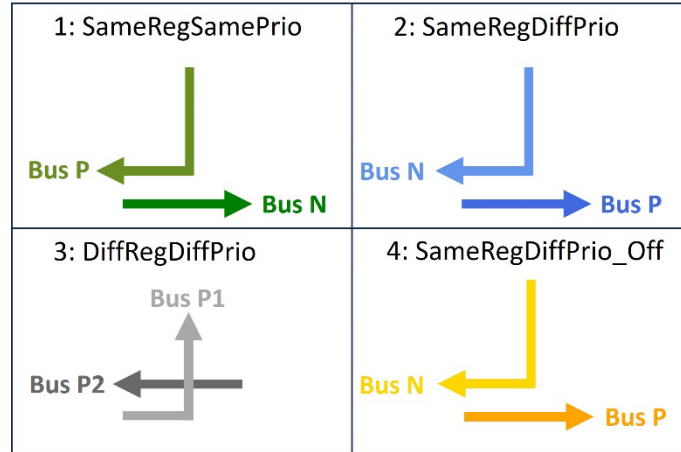


Figure 3. Overview of the Bus Routes.

The following figure 4 displays the four simulation scenarios, with three occurring during the peak hour from 7:00 to 8:00, and the fourth scenario in the off-peak hour from 10:00 to 11:00. When buses register at the same time (SameReg), the scenarios further differentiate based on whether they have the same or different priority levels (SamePrio/DiffPrio).

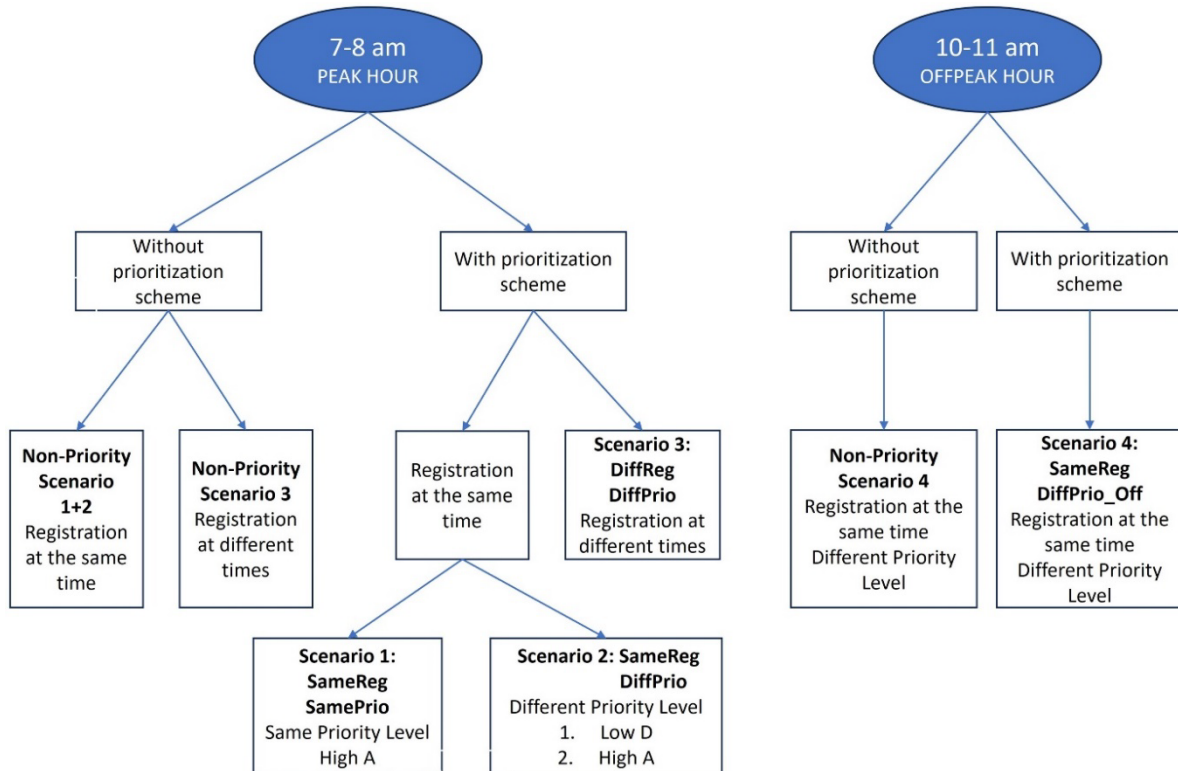


Figure 4. Overview of the simulation scenarios.

The simulation for each scenario initiates at 7:00 for the peak hour and at 10:00 for the off-peak hour, concluding at 7:40 and 10:40, respectively. The evaluation is started after a 15-minute warm-up, with data recorded at one-minute intervals. Three parameters are employed to assess the impact of prioritisation: travel time and the number of stops are measured using SUMO Multi-Entry-Exit Detectors, while queue lengths on the approaches are estimated with SUMO Lanearea Detectors [39], [40]. All detectors initiate measurements at the upstream intersection, or at the upstream bus stop in case of the eastern approach. The measurement of the queues is not extended further upstream in order not to distort the data with queues from vehicles stopping at bus stops or unsignalized intersections. The Lanearea Detectors used, deployed to measure queue length in terms of number of halting vehicles on each approach, are made up of four detectors for each approach, covering lanes leading towards the intersection. The total number of halting vehicles for one approach is estimated by summing up the values recorded by these four detectors.

For determining a statistically reliable number of simulation runs, the parameter number of stops was used, applying the t-distribution, a confidence interval of 1 and a confidence level of 99.5 %. This resulted in a number of 13 simulation runs with different seeds each for every scenario [41]. The parameter number of stops was preferred over queue length measurements because the queue length does not refer directly to the buses and their prioritisation but instead measures the effect it has on the approaches.

4. Results

The subsequent section will compare the four prioritisation scenarios and their most significant reductions and increases in the three evaluation parameters measured on the approaches to the intersection, considering both the buses and all other vehicles.

4.1 Travel Time and Duration of Prioritisation

Figure 4 shows bar plots for the mean vehicle travel time and its distribution in each scenario. The prioritised bus with the fastest average travel time is found in scenario 1. In this scenario, bus P, approaching from the northern direction and receiving B-level prioritisation, achieves a travel time of 7.84 seconds, which is a -87.86 % reduction in travel time compared to the non-priority scenario. The second most significant reduction in travel time occurs in scenario 4, where bus P, arriving from the west and granted A-level prioritisation, experiences a change in absolute travel time of -41.28 seconds, representing a -75.30 % change. However, these notable reductions in travel time for prioritised buses lead to the most considerable increases in travel time for other buses in the same scenarios. The highest percentage change between prioritised and non-prioritised runs is observed in scenario 4 for bus N, requiring 18.93 seconds longer to reach the intersection, signifying a 126.90 % increase.

The largest reduction for a non-prioritised bus occurs in scenario 2, where bus N witnesses a -66.54 % change with a travel time reduction of -43.02 seconds. In comparison, scenario 2 provides considerably less advantage to the prioritised vehicle, which, in both scenarios 2 and 4, originates from the western approach. However, scenario 4 unfolds at a different time of day, featuring different traffic patterns throughout the network. Interestingly, while scenario 2 does not enhance the travel time for the prioritised bus, it does not worsen it either, and it benefits the non-prioritised bus.

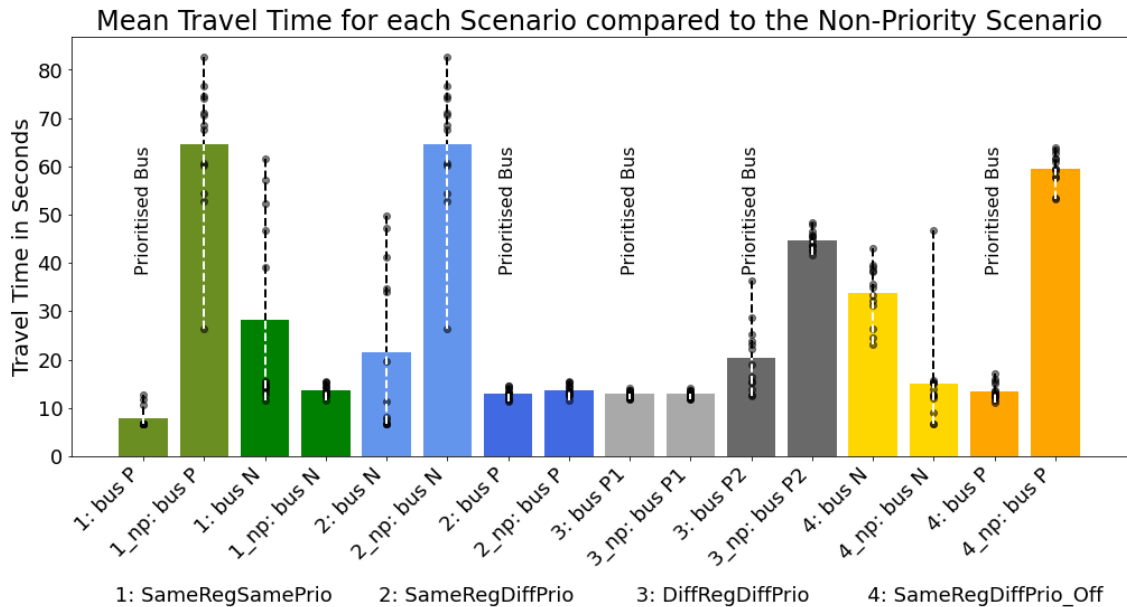


Figure 5. Mean travel times for scenario 1-4.

The most substantial reductions due to prioritisation are observed with the two highest prioritisation degrees. In scenario 4, the prioritisation procedure commences, on average, 81.31 seconds before the bus reaches the intersection, while the lesser prioritisation in scenario 1 starts, on average, 21.92 seconds before arrival. These values align closely with the ideal starting points according to prioritisation logic. A-level prioritisation ideally begins as soon as possible after the request is confirmed, and B-level prioritisation ideally starts 20 seconds before reaching the intersection. In scenario 2, prioritisation starts, on average, 98.77 seconds before the bus reaches the intersection, slightly earlier compared to scenario 4, where the same western approach is prioritised with the same priority degree.

Scenario 3 features two separate prioritisation treatments. For bus P1 approaching from the west with a priority degree B, the treatment starts 25.62 seconds before reaching the intersection, close to the ideal starting point for the prioritisation degree. In the case of bus P2, crossing the eastern stop line with a priority degree of C, the prioritisation process starts 24.69 seconds before arrival. While the treatment for bus P1 in scenario 3 aligns well with the ideal starting point for the prioritisation degree, the prioritisation process for bus P2 begins early, with an ideal time of ten seconds before reaching the stop line. Early prioritisation can be due to traffic signal phases and the prediction of the vehicle's arrival time.

All other vehicles travel fastest in scenario 4 on the western approach with a mean travel time of 20.31 seconds. The greatest benefit is experienced by vehicles using the western approach in scenario 3 with an observed reduction in travel time of -4.40 seconds, which is equal to a -14.65 % change. Conversely, the greatest increase in travel time is observed on the southern approach with 74.58 % change and an additional 18.83 seconds on average longer.

4.2 Number of Stops

For vehicles to be counted as halting the speed threshold of 1.39 m/sec must be crossed. The buses with the most significant reduction in travel time also exhibit the highest decrease in the number of stops. In scenario 4, bus P achieves the least number of stops in the prioritised runs, with zero stops, representing a -100 % change from the previous 0.84 stops. Bus P in scenario 1, while still achieving a reduction, stops only slightly more times on average in the prioritised case, with 0.08 stops, translating to a -92.31 % change. Corresponding to the high-

est increases in travel time, non-prioritised buses in scenarios 4 and 1 show the most substantial increases in the mean number of stops with bus N in scenario 4 demonstrating the highest positive percentage change, with 44.44 % more stops in the prioritised runs compared to the non-prioritised scenarios.

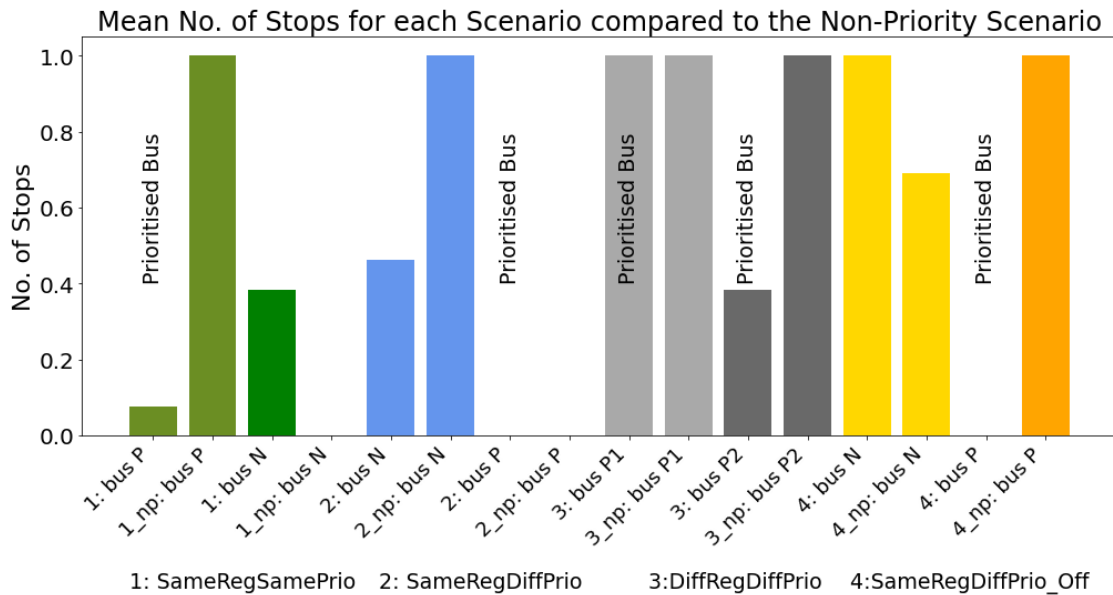


Figure 6. Mean number of stops for scenario 1-4.

For vehicles traveling through the network the western approach in scenario 4 yields the least mean number of stops at 0.34. Moreover, the western approach in scenarios 2 and 3 experiences the highest reduction in absolute value as well as percentage change, with -34.95 % and -29.11 % change, respectively. The southern approach faces the greatest disadvantages, with the highest number of stops in scenario 4 at 1.07 times on average. Additionally, scenario 4 exhibits the highest increase for vehicles on the southern approach, with an additional 0.24 stops, resulting in a 29.53 % change.

4.3 Queue Lengths

Figure 6 depicts the summed queue lengths over a 60 second period for all four scenarios on various approaches during a single simulation run. The intervals in which the prioritisation events occur are marked on the x-axis in the colour of the prioritised approach. In the first scenario the prioritisation occurs within the time interval 07:19 to 07:20 marked in green. The graphs of the prioritised northern and southern approach are at their lowest point here as shown in figure 6. The queue length values for the northern and southern approach lie below the values for this same simulation in the non-prioritised version demonstrating the prioritisation effect. In scenario 2 the minimum queue length occurs between 07:17 and 07:18 for the prioritised approaches west and east, while at the same time the non-prioritised approaches exhibit their highest queue lengths. These prioritisation effects are evident across all runs, showing the highest level of prioritisation effectively clearing queues on the approaches before the prioritised vehicle traverses the intersection. Scenario 4 also displays this prioritisation effect. During the timeframe between 10:16 and 10:18 the queues on the prioritised approaches in the west and east of the intersection are reduced to zero, while on the northern and southern approaches, the queues reach their peak. In the third scenario, two prioritisation schemes occur during the intervals from 07:16 to 07:18 marked in orange on figure 7 and between 07:21 and 07:22 marked in blue. The first prioritisation is characterised by a continuous decrease in the queue lengths observed in both the western and eastern graphs, and a sharp increase for the northern and southern graphs. The second prioritisation exhibits a less pronounced effect. However, a comparison of the mean queue length during this interval in the prioritised runs

with 56 vehicles on the western approach and 228 vehicles on the eastern approach while the non-prioritised runs exhibit a mean of 201 vehicles in the west and 494 vehicles in the east, reveals a reduction due to the prioritisation treatment and shows the presence of a green phase for the western and eastern approaches.

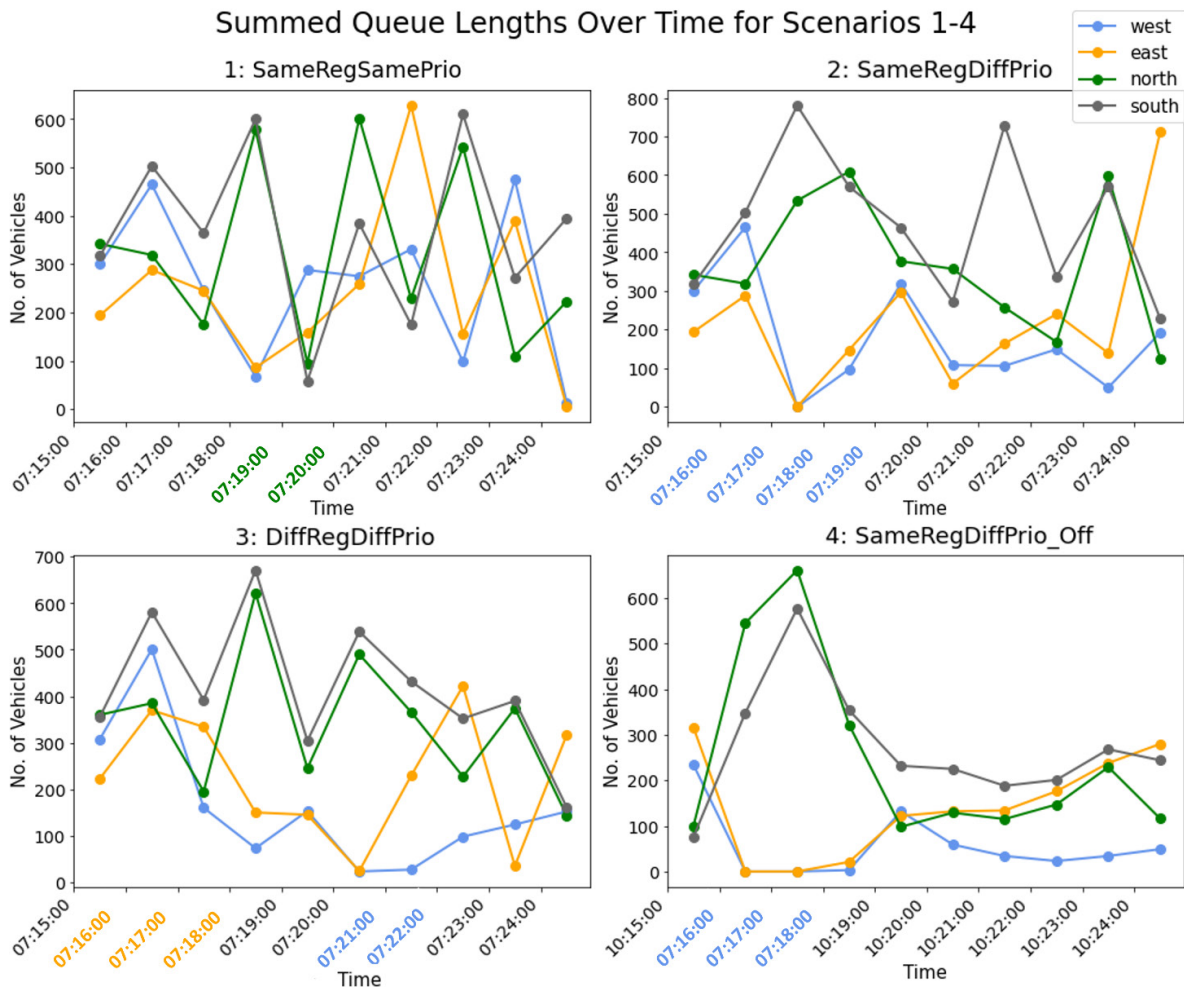


Figure 7. Summed queue lengths of one run for scenario 1-4.

Similar to the number of stops, the lowest mean values for queue length in number of vehicles per scenario are observed on the western approach, with 57 vehicles in scenario 4. The most substantial reduction in queue length is also seen on the western approach, occurring in scenario 3 with an absolute value of 43 vehicles less and in scenario 4 with a -40.51 % change. In contrast to the results on the western approach, the southern approach exhibits the highest queue length average, reaching 470 vehicles in scenario 2. The most significant increase in queue length is witnessed in scenario 4 on the same approach, with an additional 126 vehicles, representing an 89.7 % change.

5. Discussion

This chapter delves into the results presented in the previous chapter and explores the different dependencies related to prioritisation treatments. It also examines the drawbacks faced by vehicles on non-prioritised approaches and suggests potential strategies to alleviate these effects.

5.1 Influences on a Successful Prioritisation

A successful prioritisation does not solely depend on the local traffic signal phases at the designated intersection; rather, the upstream intersections before the prioritisation point play a crucial role in influencing when traffic flows onto the approaches. Scenarios 2 and 3 show that when travel time is already minimal, prioritisation may not yield further benefits. However, the prioritisation treatments in these cases do not worsen travel time or number of stops. Besides, additional green time improves travel time for vehicles on prioritised approaches. Scenario 2 also demonstrates benefits for the non-prioritised bus on the crossing approach, while scenario 4 highlights significant reductions for the prioritised bus. The most notable reduction (87.86 %) for a prioritised bus occurs in scenario 1, indicating the influence of route and timing on prioritisation success. The peak-hour scenarios showed more and more pronounced periodic peaks in their queue lengths on all four approaches independent of prioritisation treatments than the off-peak scenario. In the peak hour the stability of the queues on the four approaches is determined by a large extent by the timing of the previous traffic signals, as the traffic passing the prioritised intersection remains largely constant for each approach outside of the prioritised scenarios. The prioritisation benefits such as the trip time savings on each approach have a higher correlation to the timing of when the traffic flows onto the approach which is dictated by the previous intersections than to the edge traffic flows. This emphasises the weight the traffic flow patterns play in realising the benefits as well as creating the disadvantages for the vehicles traversing the intersection.

It may be argued that while looking at the reduction of number of stops, travel time and queues on the approaches created by the treatments, even greater benefits can be achieved when several intersections on the paths of the bus lines are able to execute prioritisation schemes. However, one intersection locally already achieves notable optimisation in travel for the chosen vehicles.

The implementation of the prioritisation is highly dependent on the prediction of arrival time at the intersection for the prioritised vehicle. The ETA-function within the code is updated every simulation second and in turn the selection of the prioritisation scheme is updated every second until 62 seconds before arrival. This procedure should ensure that the start time of the green phase for the prioritised approach falls close to the ideal starting time determined by the prioritisation degree of the vehicle. In most scenarios the starting time falls close to the ideal case, for example in scenario 1 with a prioritisation degree of B, the treatment commences on average 21.92 seconds before crossing. However, bus P2 in scenario 3 traverses the intersection 24.69 seconds after the start of the green phase, which is 14.69 seconds away from the goal for priority degree C. The historical data used in the ETA function may be improved by using more runs and a higher amount of data to predict the correct travel parameters more accurately. Nevertheless, while the prioritisation should not start too early it would be more disadvantageous for the goal of the prioritisation if it starts too late. The reliance on accurate ETA predictions is inherent for implementing bus priority based on V2X communication instead of detection points. If the ETA prediction is accurate, a well-timed prioritisation is possible, which enables a quick return to normal traffic signal operation.

5.2 Ramifications of Prioritisation and Pathways to Solutions

All scenarios demonstrate that each intervention introduces consequences for all vehicles traversing the intersection. In scenario 4, the adverse effects on the southern approach were most pronounced, leading to an increase in the evaluation parameters. It may be argued that the southern approach is more at risk for longer travel times and higher queue lengths as it is the shortest out of the four and offers less space for vehicles to queue. The longest approach, the western approach, experiences the greatest benefits, indicating that spatial constraints may influence the ability of the intersection to deal with the consequences from the prioritisation events. Scenario 4 also illustrates that the peak queues on the approaches return to pre-prioritisation levels within the subsequent two-time intervals. This temporal pattern holds true for

the other scenarios as well, indicating that compensation for the intersection can be achieved within a specific time constraint. In all scenarios, one or two isolated prioritisation events are depicted. The consequences for the approaches may differ when multiple prioritisation events occur rapidly, and compensation for the non-prioritised approaches could occur through subsequent prioritisation events. As seen in scenario 3, where two buses are prioritised separately and subsequently, there are almost no worsening effects on the evaluation parameters. The range of increase in the evaluation parameters is much less than observed in scenario 4 or 2, where in both cases only one prioritisation event occurs.

One studied approach for reducing the effects on the opposing non-prioritised approaches is to introduce compensation phases, where the duration of green time on those approaches is extended. However, according to Collier [23], such measures can cause additional delay. In the simulated logic for this project, after prioritisation ends, the phases for the opposing approaches are not extended. Instead, an immediate switch is made to those phases with consideration of minimum green times. The cycle itself is never broken up to avoid additional longer intergreen times. This measure itself may also contribute to diminishing the adverse prioritisation effects.

5.3 Extension of Prioritisation Logic

In scenario 1, both buses request prioritisation at the same second and with the same priority level, A1. However, only bus P receives prioritisation. In all other cases where the prioritisation degrees are different between two or more buses, only the bus with the highest level is treated to prioritisation. The code generates a list of buses that are present in the network and approaching the intersection with the help of a TraCI-function, which lists all objects present in the network sorted in alphabetical and numeric order. Consequently, bus P is higher in the network list of buses and therefore selected as the prioritised bus since it is listed before bus N when the two have the same priority degree. If bus P were named behind bus N in numerical order bus N would have received prioritisation. In this case of the same exact priority level, it would be useful to develop a second decision level to ensure that the bus with the highest occupancy or delay value is selected instead of relying solely on alphanumeric order. It should be noted, however, that the logic still prioritises the bus with the highest degree in any case.

6. Conclusion and Recommendations

This paper proposes a bus prioritisation logic for signalised intersections that is based on bus delay and occupancy instead of the prevailing first-come, first-served strategy. For buses, detection and data transmission via V2X communication is assumed, as this is expected to increasingly find its way from research into reality in the coming years, especially in the area of bus prioritisation. The basis for prioritisation is an accurate Estimated Time of Arrival (ETA) estimation that ensures an optimal timing for the start of the prioritisation treatment. The prioritisation hierarchy guarantees that vehicles with the highest priority levels receive tailored acceleration in travel time, based on their delay and occupancy values, rather than the timing of their registration at the intersection. The priority levels are segmented into four degrees, allowing for differentiation in prioritisation measures by specifying the commencement of the desired green time, each with varying effects on crossing approaches. When choosing treatments at the traffic signal, three options—green extension, red truncation and red truncation expanded—are evaluated. The selected option is the one that aligns the start of the green time closest to the ideal time specified by the priority degree, further ensuring a precise implementation of bus prioritisation measures. The developed bus prioritisation logic is evaluated in a variety of scenarios for microscopic simulation, which investigated in particular, what happens if several buses with different priority levels request prioritisation at the same time.

The simulations of prioritisation procedures revealed significant improvements for most prioritised buses, resulting in reductions of up to over 85 % in mean travel time and, in some cases, eliminating stops altogether. Among the five prioritised buses, three experienced substantial benefits from the prioritisation treatment. The other two buses, already operating at minimum travel times and stops, did not witness further reductions nor increases. Moreover, vehicles on prioritised approaches saw mostly positive impacts. The most significant improvements in travel time, stops, and queue length on the prioritised approach were observed with the highest prioritisation degrees A and B, exemplified in scenarios 1 and 4.

A pivotal outcome is revealed through the simulations: the efficacy of the developed bus prioritisation algorithm, manifesting in significant reductions in travel time for buses that are already delayed. Such delays often coincide with high demand traffic flows. This highlights the importance of prioritising buses based on existing delays - an approach that can effectively realign buses with their schedules, which is particularly beneficial in high occupancy scenarios where many people can gain valuable time savings. If two buses arrive at the same time, those with lower priority levels experience no or less intervention, but the deterioration in their travel time is never as great as the improvement for the high-priority buses.

All prioritisations are interventions in the regular traffic signal cycle, changing the lengths of the green phases to benefit certain approaches. These interventions also caused consequences for all vehicles that were travelling on the non-prioritised roadways. The highest increases in the evaluation parameters were observed on the approach south of the intersection, which has the shortest length out of the four. The highest increases in travel time for the non-prioritised buses were found in the same scenarios where the prioritised buses had the highest decreases. However, when two buses were prioritised separately after each other, the evaluation parameters for the non-prioritised approaches were not as increased compared to the scenarios where only one bus received priority. This finding encourages further simulations with more than one bus being prioritised, extending the time constraints of the simulated scenarios, and measuring the effects several subsequent prioritisation events have on the evaluation parameters.

The effectiveness of the prioritisation treatments was found to be reliant on several factors such as the location of and the traffic signal phases of neighbouring intersections, which in turn heavily influence the inflow of traffic onto the approaches of the prioritised intersection. The inclusion of other intersections along a bus's route into the prioritisation would greatly advance the benefits for the delayed bus. This aspect can also be further explored in future studies.

Notably, travel patterns across the network exhibited substantial variations between peak and off-peak hours, suggesting the need to study an extension of prioritisation treatments to more hours of the day. This could prompt a study into optimal conditions for prioritisation, aiming to discern when it most effectively enhances the overall performance of an intersection.

The initiation logic of prioritisation depends on the accurate prediction of the vehicle's arrival time at the intersection. While generally prioritisations started close to their ideal times, further improvement in this function may yield benefits. The combination of historic and current on-line data aimed at creating an accurate travel time prediction process on which prioritisation timings can depend, may also be of interest for further studies as it is relevant to the implementation of future highly detailed responsive prioritisation processes using V2X technologies.

The prioritisation logic is centred around delay parameters, utilising occupancy information to determine rankings within the delay levels. Scenario 1 highlighted the need for establishing a secondary level of prioritisation logic. The prioritisation logic, levels, and degrees are adaptable to new data, allowing for further adjustments in ideal start times of treatments. This flexibility enables the degree of intervention to be extended or lessened as needed. Further research could compare which parameters – occupancy or delay, and which combination

of prioritisation degrees results in greater reliability for the bus services and yet mitigates the effects on all other vehicles the most.

The comprehensive examination of various simulation scenarios showed the benefits of bus prioritisation treatments based on delay and occupancy data. The findings reveal that a high reduction in travel time and number of stops the buses experience on the approach to the intersection can be achieved with a precise intervention into the traffic signal cycle to advance the desired green phase. The use of delay and occupancy data in a future scenario with V2X communication showed the potential to effectively reduce delays, especially for buses with high occupancy, and thus improve the reliability and attractiveness of public transport.

Data Availability Statement

The SUMO simulation network of Ingolstadt which was the basis for this paper's simulations can be assessed on GitHub [36].

Competing interests

The authors declare that they have no competing interests.

Funding

This work has been carried out as part of the KIVI research project (Artificial Intelligence in the Transportation System of Ingolstadt), which is funded by the German Federal Ministry of Transport and Digital Infrastructure (Funding ID: 45KI05A011).

Author Contributions

Katia Juliane Schmidt: Methodology, Writing – original draft, Natalie Steinmetz: Supervision, Writing – review & editing, Martin Margreiter: Supervision, Writing – review & editing

References

- [1] Stadt Ingolstadt, "Stadt Ingolstadt Verkehrsentwicklungsplan 2025," Aug. 2017. Accessed: Jan. 05, 2024. [Online]. Available: https://www.ingolstadt.de/media/cus-tom/2789_1089_1.PDF?1540885166. [Accessed: 17-January-2024]
- [2] M. Ilic, N. Sautter, M. Margreiter, A. Loder, K. Bogenberger, "Revisiting Urban Traffic Control in Austria, Germany and Switzerland - Requirements for Future Developments", *Transportation Research Board - 101st Annual Meeting*, 2022.
- [3] Ilic, M.; Sautter, N.; Margreiter, M.; Loder, A.; Kessler, L.; Kutsch, A.; Bogenberger, K.: "Strategien urbaner Verkehrssteuerung in der DACH Region - Anforderungen an künftige Entwicklungen," *KIVI Konsortialtreffen*, 2022.
- [4] M. Margreiter, S. Krause, H. Twaddle, J. Lüssmann, "Evaluation of Environmental Impacts of Adaptive Network Signal Controls Based on Real Vehicle Trajectories", *Transportation Research Procedia*, 2014.
- [5] C. Diakaki, M. Papageorgiou, V. Dinopoulou, I. Papamichail, and M. Garyfalia, "State-of-the-art and -practice review of public transport priority strategies," *IET Intelligent Transport Systems*, vol. 9, no. 4, pp. 391–406, May 2015.
- [6] H. R. Smith, B. Hemily, M. Ivanovic, G. Fleming, Department of Transportation, and ITS America, *Transit Signal Priority (TSP): A Planning and Implementation Handbook*, 2005. [Online]. Available: https://nacto.org/wp-content/uploads/2015/04/transit_signal_priority_handbook_smith.pdf. [Accessed: 10-February-2024]

- [7] Y. Lin, X. Yang, N. Zou, and M. Franz, "Transit signal priority control at signalized intersections: a comprehensive review," *Transportation Letters: The International Journal of Transportation Research*, vol. 7, no. 3, pp. 168–180, Dec. 2014.
- [8] NACTO, "Active Transit Signal Priority | National Association of City Transportation Officials," *National Association of City Transportation Officials*, May 05, 2016. [Online]. Available: <https://nacto.org/publication/transit-street-design-guide/intersections/signals-operations/active-transit-signal-priority/>. [Accessed: 10-February-2024].
- [9] R. Machemehl, "Class Notes: CE 391 L, Advanced Traffic Engineering," the University of Texas at Austin, season-01 1996.
- [10] NACTO, "Queue Jump Lanes | National Association of City Transportation Officials," *National Association of City Transportation Officials*, May 05, 2016. [Online]. Available: <https://nacto.org/publication/transit-street-design-guide/intersections/intersection-design/queue-jump-lanes/>. [Accessed: 05-February-2024]
- [11] P. G. Furth and J. L. SanClemente, "Near side, far side, uphill, downhill: Impact of bus stop location on bus delay," *Transportation Research Record*, vol. 1971, pp. 66–73, Jan. 2006.
- [12] M. Garrow and R. B. Machemehl, "Development and evaluation of transit signal priority strategies," *Journal of Public Transportation*, vol. 2, no. 2, pp. 65–90, Jun. 1999.
- [13] NACTO, "Transit Signal Progression | National Association of City Transportation Officials," *National Association of City Transportation Officials*, May 05, 2016. [Online]. Available: <https://nacto.org/publication/transit-street-design-guide/intersections/signals-operations/transit-signal-progression/>. [Accessed: 23-February-2024]
- [14] T. Urbanik and R. W. Holder, *Evaluation of alternative concepts for priority use of urban freeways in Texas*, 1977.
- [15] FTA, "Signal priority," *Federal Transit Administration*, Dec. 06, 2015. [Online]. Available: <https://www.transit.dot.gov/research-innovation/signal-priority>. [Accessed: 08-February-2024]
- [16] W. Ma and Y. Bai, "Serve Sequence Optimization Approach for Multiple Bus Priority Requests Based on Decision Tree," *Proceedings of the Seventh International Conference of Chinese Transportation Professionals Congress*, Mar. 2008.
- [17] Q. He, L. Head, and J. Ding, "Heuristic algorithm for priority traffic signal control," *Transportation Research Record*, vol. 2259, no. 1, pp. 1–7, Jan. 2011.
- [18] E. R. Allsop, Ed., "Priority for Buses at Signal-Controlled Junctions: Some Implications for Signal Timings," *Proc., 7th International Symposium on Transportation and Traffic Flow Theory*, Aug. 1977.
- [19] H. K. Evans and G. W. Skiles, "Improving Public Transit Through Bus Preemption of Traffic Signals," *Traffic Quarterly*, vol. 24, no. 4, Oct. 1970, [Online]. Available: <https://trid.trb.org/view/115793>. [Accessed: 11-February-2024]
- [20] S. Gallivan, C. P. Young, and J. R. Peirce, "Bus priority in a network of traffic signals," Paris, France. 1980.
- [21] T. Y. El-Reedy and R. Ashworth, "The effect of bus detection on the performance of a traffic signal controlled intersection," *Transportation Research*, vol. 12, no. 5, pp. 337–342, Oct. 1978.
- [22] B. R. Cooper, R. A. Vincent, and K. Wood, "Bus-actuated traffic signals-initial assessment of part of the Swansea bus priority scheme," Report LR 925, Jan. 1980, [Online]. Available: <https://trid.trb.org/view/156063>. [Accessed: 23-February-2024]
- [23] J. C. Collier, "Bus selective detection in Swansea," Traffic Advisory Unit Report, Jan. 1980, [Online]. Available: <https://trid.trb.org/view/197855>. [Accessed: 23-February-2024]
- [24] G. T. Bowen, R. Bretherton, J. R. Landles, and D. Cook, "Active bus priority in SCOOT," *Seventh International Conference on Road Traffic Monitoring and Control*, pp. 73–76, Jan. 1994.
- [25] N. B. Hounsell, F. McLeod, R. Bretherton, and R. Bowen, "PROMPT: field trial and simulation results of bus priority in SCOOT," *8th International Conference on Road Traffic Monitoring and Control*, pp. 90–94, Jan. 1996.

- [26] C.-F. Liao and G. A. Davis, "Field Testing and Evaluation of a Wireless-Based Transit Signal Priority System," Digital Conservancy, Oct. 2011, [Online]. Available: <https://conservancy.umn.edu/bitstream/11299/117369/1/CTS%2011-25.pdf>. [Accessed: 23-February-2024]
- [27] L. Head, D. Gettman, and Z. Wei, "Decision model for priority control of traffic signals," *Transportation Research Record*, vol. 1978, no. 1, pp. 169–177, Jan. 2006.
- [28] G. Chang, M. Vasudevan, and C. C. Su, "Modelling and evaluation of adaptive bus-preemption control with and without Automatic Vehicle Location systems," *Transportation Research Part A: Policy and Practice*, vol. 30, no. 4, pp. 251–268, Jul. 1996.
- [29] Y. Lin, X. Yang, G. Chang, and N. Zou, "Transit Priority Strategies for Multiple Routes under Headway-Based Operations," *Transportation Research Record*, vol. 2366, no. 1, pp. 34–43, Jan. 2013.
- [30] M. Gay et al., "Nutzung der C2X-basierten ÖV-Priorisierung an signalisierten Knotenpunkten," Bundesanstalt für Straßenwesen, Bergisch Gladbach, Germany, Verkehrstechnik Heft V 353, Jan. 2022. [Online]. Available: <https://bast.opus.hbz-nrw.de/opus45-bast/frontdoor/deliver/index/docId/2595/file/V353+BF+Gesamtversion.pdf>. [Accessed: 23-February-2024]
- [31] C. Ebert, "Connected Car und Car2x Communication | Institut für Automatisierungstechnik und Softwaresysteme | Universität Stuttgart," Universität Stuttgart, Feb. 01, 2018. [Online]. Available: <https://www.ias.uni-stuttgart.de/service/begriffslexikon/connected-car-und-car2x-communication/>. [Accessed: 23-February-2024].
- [32] GEVAS, "CCall," 2021. [Online]. Available: <https://www.gevas.eu/losungen/ccall>. [Accessed: 23-February-2024].
- [33] P. Kuchár, R. Pirník, A. Janota, B. Malobický, J. Kubík, and D. Šišmišová, "Passenger Occupancy Estimation in Vehicles: A review of current methods and research challenges," *Sustainability*, vol. 15, no. 2, p. 1332, Jan. 2023.
- [34] ETSI, "ETSI TS 103 301 V2.1.1 (2021-03)," European Telecommunication Standards Institute, France, RTS/ITS-001948, 2021. [Online]. Available: <http://www.etsi.org/standards-search>. [Accessed: 26-April-2024]
- [35] "Reallabor High Definition Testfeld," *Technische Hochschule Ingolstadt*, Jul. 18, 2023. [Online]. Available: <https://www.thi.de/forschung/carissma/labore/reallabor-hdt/>. [Accessed: 23-February-2024]
- [36] M. Langer, M. Harth, J. Brodersen, "GitHub - TUM-VT/sumo_ingolstadt: Traffic simulation of Ingolstadt in SUMO," *GitHub*, Aug. 2023. https://github.com/TUM-VT/sumo_ingolstadt. [Accessed: 13-February-2024]
- [37] Harth, M., Langer, M., & Bogenberger, K. (2022). "Automated Calibration of Traffic Demand and Traffic Lights in SUMO Using Real-World Observations," *SUMO Conference Proceedings*, 2, 133–148.
- [38] "Lichtsignalanlagen," Rsa-online, Jun. 2015. [Online]. Available: <https://www.rsa-online.com/15/Lichtsignalanlagen/Lichtsignalanlagen.html>. [Accessed: 13-February-2024]
- [39] P. Á. López et al., "Microscopic Traffic Simulation using SUMO," SUMO-Multi-Entry-Exit Detectors, 2024. [Online]. Available: https://sumo.dlr.de/docs/Simulation/Output/Multi-Entry-Exit_Detectors_%28E3%29.html. [Accessed: 13-February-2024]
- [40] P. Á. López et al., "Microscopic Traffic Simulation using SUMO," SUMO-Lanearea Detectors, 2024. [Online]. Available: https://sumo.dlr.de/docs/Simulation/Output/Lanearea_Detectors_%28E2%29.html. [Accessed: 09-February-2024]
- [41] J. Puhani, *Kleine Formelsammlung zur Statistik*. 2020.