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Implementing an Approximation of Cumulative Prospect Theory into Mixed Linear Programming – an Application to Bio-Economic Modelling at Farm-Scale Considering Crop Insurance

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Abstract

Many empirical studies have found Cumulative Prospect Theory (CPT) superior in depicting risk behavior compared to the expected utility approach and literature now offers also CPT related parameter estimates for European farmers. CPT combines two segments of utility functions, a convex, risk loving one for losses and a concave, risk averse one for gains, and assigns subjective weights to the pay-offs according to their cumulative probabilities. So far, no implementation of CPT into constrained optimization problems exists, allowing for instance, the simulation of risk management under CPT in farm-scale programming models. To close this gap, we propose to combine endogenous sorting of the pay-offs based on integer variables with a piece-wise linear approximation of the value function using SOS2 (Special Ordered Sets of Type 2) variables. The SOS2 variables are required to deal with the convexity of the loss segment of the utility function. The integer sorting assigns the weights to the pay-offs according to their cumulative probabilities, it requires that all pay-offs are equally likely. Simulating optimal uptake levels of variants of a hypothetical crop insurance product with an evolved bio-economic model at farm-scale serves a proof of concept. The model considers adjustments in the cropping plan and allows for partial insurance coverage, in opposite to existing studies which evaluate the uptake of crop insurance at fixed crop choices and depict coverage as a yes-no decision.

The approximation error of the approach is found as negligible small and the numerical burden compared to optimization under risk neutrality as still acceptable. The proposed approximation approach is quite general and applicable for any utility function increasing in the pay-off value and does not require its differentiability. It can also be applied without probability weighting. The empirical application underlines that the approach generates the expected behavior when a risk reducing strategy, here crop insurance, is considered under CPT. Insured acreage generally increases with higher strike levels where more frequently occurring but lower crop damages are covered, and with reduced cost of the insurance products. Using crop insurance as a risk management strategy is found to interact with other measures such as adjustments in cropping shares. This underlines the usefulness of an approach which allows to optimize interacting risk management strategies at farm-scale under CPT, considering resource and other relevant constraints.

Keywords

Cumulative Prospect Theory; constrained optimization; Mixed Integer Programming; bio-economic farm-scale modeling; crop insurance

1 Introduction

Cumulative Prospect Theory (CPT, TVERSKY and KAHNEMANN, 1992, in the following abbreviated as TK) is more flexible in depicting risk utility compared to expected utility theory and was found to describe more accurately risk behavior in many empirical studies (cf. WAKKER, 2010). The risk utility function in Prospect Theory is convex for losses and concave for gains around a given reference point. This implies risk loving for losses and risk aversion for gains, in contrast to the standard assumption of expected utility (EU) theory where the risk utility function is concave over the range of considered pay-offs. CPT adds cumulative probability weighting such that the more extreme gains or losses receive higher weights compared to their objective probability. This effect gets stronger the lower the attached objective probabilities to such outcomes.

There is a growing body of literature which estimates parameters related to CPT, also for European farmers (BOUGHERARA et al., 2017; BOCQUÉHO et al., 2014; COELHO et al., 2012). Existing literature evaluates risk management options under CPT keeping the

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farm program fixed as no consistent incorporation of CPT into constrained optimization approaches is available. Instead, agricultural programming models consider risk behavior mostly by maximizing a weighted sum of the expected returns and their variance (E-V, also called Mean-Variance, M-V). The use of a linear approximation of E-V called MOTAD (Minimization of Total Absolute Deviations, HAZELL, 1971) in models based on Mixed Integer Programming (MIP) is also common. The E-V approach is consistent only under normally distributed pay-offs or under a quadratic utility function. Curvature requirements in constrained optimization exclude the consideration of risk loving under E-V. Moreover, a concave quadratic utility function implies increasing relative and absolute risk aversion in wealth, which is unwelcome in empirical applications.

Against this background, an implementation of CPT into programming models is desirable, but also challenging. First, both the risk utility function and the weighting function for the probabilities are neither linear nor quadratic. This excludes their direct use in models which are based on MIP or mixed integer quadratically constrained programming, the most widespread solutions to capture indivisibilities, returns-to-scale or other convex relations as well as if-conditions in constrained economic maximization. Second, the non-concavity of the utility function on the loss segment excludes the use of a large set of widely used, gradient based solvers for non-linear programming. Third, the probability weighting requires an ordering of the pay-offs which are endogenous variables (cf. HENS and MEYER, 2014), which is possible only based on integer variables. It, therefore, comes at little surprise that the body of literature to optimize risk management under CPT is limited.

The empirical example is taken from agricultural economics where the use of so-called bio-economic models is quite common (BRITZ et al., 2012; JANSSEN and VAN ITTERSUM, 2007). These models are often based on MIP, as it the case of the FarmDyn model (LENGERS et al., 2013) employed in the empirical application. It is used here to simulate the crop specific coverage level of different variants of a hypothetical crop insurance product based on realized regional yields, considering simultaneously adjustments in the crop mix. Analyzing crop insurance is motivated by a lively discussion around insuring weather risks in agriculture (cf. ODENING and SHEN, 2014). This paper is organized as follows. After reviewing existing literature, the implementation is detailed, followed by a section on the empirical application. Their combination then feeds in the discussion section and a brief summary.

2 Literature Review

Many studies have found farmers to be risk averse (IYER et al., 2020) which motivates optimizing expected risk utility in farm-scale programming models (cf. JANSSEN and VAN ITTERSUM, 2007). But newer studies suggest that farmers rather decide according to CPT (BOUGHERARA et al., 2017, BOCQUÉHO et al., 2014, COELHO et al., 2012), which can imply risk aversion or risk loving, depending on whether pay-offs refer to gains or losses around a reference point. No consistent implementation of CPT in a constrained optimization set-up exists which motivates this paper.

Besides market and policy risks faced by firms in all sectors, agriculture is also subject to considerable production risk due to weather, pests and diseases. While market-based instruments against price risk are existent and frequently used by farmers, insurance against weather risks are hardly taken up if not subsidized (ODENING and SHEN, 2014). This explains why subsidized crop insurance exists in many countries, such as for decades in the US. Member States of the European Union can choose it as a measure under the Common Agricultural Policy, an option chosen, for instance, in Spain. Therefore, assessing the uptake of crop insurance provides an inviting case for our empirical application as it asks for considering risk and risk behavior.

In Germany, non-subsidized hail insurance is widespread. It is based on on-site assessment of the damage which comes along with considerable transaction costs. Its frequent update by farmers despite these higher transaction costs might reflect the combination of a low probability and high damages of the hail risk, a combination receiving a high weight under CPT. Hail damage might in future be assessed based on satellite images (BELL and MOLTHAN, 2016) which reduces transaction cost. The extension of crop insurance to weather events with more moderate damages, such as period of droughts, is challenging. If they require on-site inspection, the transaction costs to inspect the damage are high compared to paid-out indemnities. Furthermore, determining the actual damage caused by such weather events asks for an estimate of the site-specific yield which would have occurred without the insured event. This requires considering the realization of all other stochastic sources of crop damages, such as pests or other climatic variables. Therefore, so-called index-based crop insurances are developed (cf. DALHAUS et al., 2020). They pay out indemnities solely based on the occurrence of an insured weather event, which excludes moral hazard. The latter challenges insurance coverage of realized site-specific yields which depend also on hard to monitor farm management decisions. But index-based crop insurance can come along with considerable basis risk due to often moderate correlation, only, between the insured weather events and overall crop yield variability (e.g. WEBBER et al., 2018).

An alternative to index-based products is the use of realized regional yields which is common in the US. This clearly also implies basis risk (FINGER, 2012). BABCOCK (2015) determines at given cropping program optimal uptakes rates of a subsidized crop insurance under CPT considering different strike levels, i.e. the minimal yield loss covered by the insurance. The insured loss is determined by deviations of regional yields from trends, as in the hypothetical example analyzed here. DALHAUS et al. (2020) design weather index-based insurance contracts for winter wheat producers in Eastern Germany considering CPT and EU under different risk-behavioral parameters taken from literature. They evaluate different contract variants which differ in strike levels and whether premiums are paid only in years with no losses. They find that contract variants leading to the highest uptake rates can differ depending on whether EU or CPT is assumed, and develop a contract design which benefits both EU and CPT maximizers. As in BABCOCK (2015), the farm program is taken as given. The approach chosen here allows instead both for an endogenous choice of the insured crops and their insured area shares, and considers simultaneously adjustments in the farm program. This requires an optimization approach. CAO et al. (2020) apply the logic of a CPT framework where gains and losses are treated differently in econometric work to explain why beef farmers exit or stay in subsidized margin insurance in Canada. Due to specific program design, farmers might (also falsely) expect gains such that certain participation decisions can be rather seen an investment strategy.

Existing approaches which do not evaluate a yes-no decision under CPT, but instead optimize a risk management strategy stem mostly from finance. In order to optimize a portfolio under CPT, HENS and MAYER (2014) propose to first define the efficient M-V frontier of the prospects to consider. This frontier depicts for the different given mean returns, as found in the considered prospects, the prospect with the lowest possible variance in returns. Only considering prospects on the frontier considerably reduces the prospects to consider in further analysis. In their paper, these prospects are assets in a classical portfolio selection problem defined by their pay-offs in distinct futures. Accordingly, the sole variable of choice to optimize are the shares of these assets, and no further constraints are considered. Their approach requires to generate first a large set of M-V optimal solutions which reflect different compositions of the portfolio. Afterwards, the risk utility function is evaluated for each considered composition and future, the resulting pay-offs are ordered to determine their cumulative probabilities and to calculate the attached subjective weights. This allows to quantify the utility attached to each considered portfolio choice. Afterwards, the best of the portfolios is selected. This iterative approach is clearly not easily applicable to constrained optimization where portfolio choice is computationally much more demanding. The work of HENS and MEYER (2014) builds on LEVY and LEVY (2004) which compare CPT to M-V analysis. They show that CPT efficient portfolios composed of shares of risky assets almost coincide with the optimal ones under M-V if each asset has normally distributed pay-offs, shares of the assets can be freely chosen and the assets are not perfectly correlated. They prove formally that the CPT efficient frontier is a sub-set of the MV-efficient one under these conditions.

CONSIGLI et al. (2019) solve the same problem as the two paper above by optimizing shares of assets in a portfolio for which the distribution of the returns for each asset is given. As HENS and MEYER (2014), they first generate random samples of such portfolios where each sample comprises different weights of the assets, and then select the best ones in an unconstrained optimization problem under CPT. They propose to use cubic splines to solve the problem of finding the best solution, given the non-differentiability and non-concavity of the function to optimize.

The approaches from finance to find CPT optimal portfolios are not applicable to farm-scale optimization even if the decision variables are crop shares, only, if besides adding up of the crop acreages to total land also other constraints are considered. These relate, for instance, to other limiting resources such as labor or machinery, to crop rotational limits or to legal restrictions. This motivates the search for an approach to implement CPT into constrained optimization problems.

COEHLO et al. (2012) provide the only paper found which applies CPT in a constrained optimization set-up where other constraints beside adding up of shares are considered. As in the empirical part of this paper, CPT is discussed in the context of farm-scale optimization. Their CPT implementation seems however not consistent in two aspects. First, the weighting of each future requires the cumulative probabilities according to the order of the pay-offs, which can change during optimization. This does not fit to their use of fixed weighting coefficients for each future. Second, they use fixed certainty equivalents per unit of net revenue in the different futures. This seems inconsistent as certainty equivalents before weighting depend under CPT on the non-linear risk utility function. Against this background, the approach proposed here seems the first consistent implementation of CPT into a constrained optimization set-up.

3 Methodology

The approach combines three elements. The first is the estimation of a piece-wise linear approximation of the risk utility function. Second, this step-wise approximation is integrated into the constrained optimization problem based on a set of distinct futures. Third, the pay-offs are endogenously sorted in the model to assign them weights based on their cumulative probabilities. How these elements are combined is discussed then in a follow-up section on the solution strategy.

3.1 Piece-Wise Approximation of the Utility Function

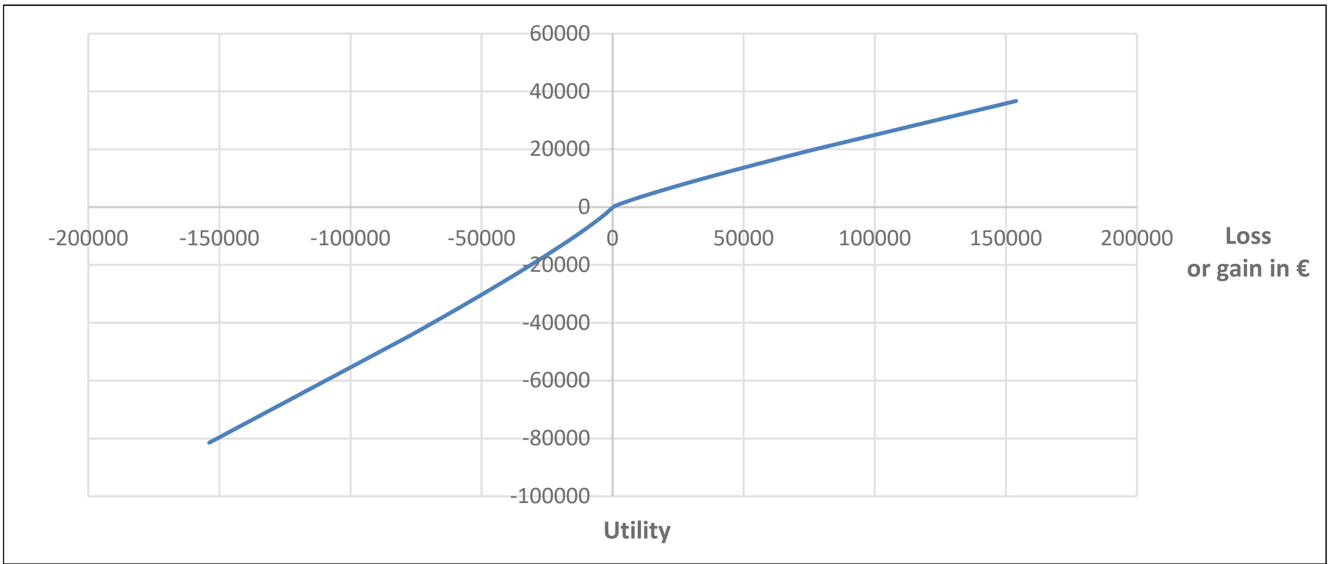
The approach uses the TK type of utility function which combines segments of two power utility functions which relate to gains, first line in Equation (1), and losses (second line). Utility is defined based on a variable x. It depicts the pay-off to evaluate minus the pay-off at a reference point rp such that positive x depict gains and negative x losses:

$$u(x) = \begin{cases} x^{\alpha} & x \geq 0 \\ -\gamma(-x)^{\beta} & x \leq 0 \end{cases} \quad (1)$$

The function can accommodate different exponents α for gains and β for losses, typically restricted to the range between 0 and 1. It turns the exponential curve for losses relative to the one for gains based on the positive parameter γ (see also Figure 1). This parameter γ was typically found to be larger than unity in empirical studies such that losses receive a higher weight than gains of the same absolute size, even if α is equal to β. Due to $\beta < 1$ and the negative sign before γ, the loss segment is convex which implies risk loving. The segment for gains is concave, implying risk aversion, as typically assumed under EU both for gains and losses.

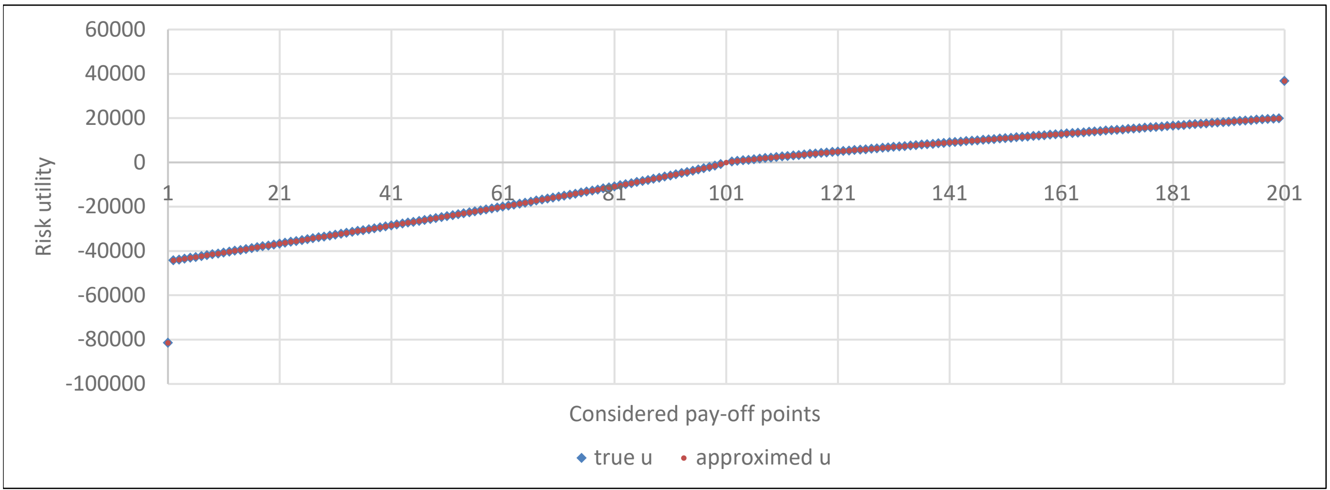
In order to define a piece-wise approximation of the TK risk utility function, the range of pay-offs to consider is needed. To do so, let s define the observed or estimated spread of the pay-offs. Here, *n-2* equally

Figure 1. TK risk utility function as used in the empirical analysis



Source: Authors

Figure 2. Piece-wise linear approximation of the TK utility function



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distanced ordered outcomes of x on the range $[-s/2 + rp, +s/2 + rp]$ are used. Their spread is proposed to be the larger of 2 rp or the difference between the minimum and maximum of the empirically observed pay-offs. As the pay-offs under CPT are endogenous and known after optimization, only, the spread of the optimized risk-neutral pay-offs x_f^* is used instead here, their mean defines the reference point *rp*. This follows the argumentation of KŐSZEGI and RABIN (2007).

In order to allow for potential changes in the minimum and maximum pay-offs when later maximizing risk-utility, two points at $-s + rp$ and $+s + rp$ are added. This results in *n* ordered pay-offs for which the risk utility function $u(x)$ is evaluated. The example application uses the original parameters of TK 1992, i.e. $\alpha = \beta = 0{,}88$ and $\gamma = 2{,}22$ which results in the curve shown below in Figure 1. The graphic also depicts the considered spread of the pay-offs in €. As seen, the two segments are only moderately bended for larger gains or losses as the exponents of the power utility functions are close to unity. What is more apparent is the different slope of the loss segment due to a larger γ.

The first derivatives fd of this risk utility function are approximated at each endogenously optimized approximation point x_i based on:

$$fd_i = \frac{u(x_i) - u(x_{i-1})}{x_i - x_{i-1}} \tag{2}$$

The number of approximation points *n* which are later introduced in the simulation framework is exogenously given, and in here chosen as 50. In order to find optimal values of these approximation points, a larger number of given payouts $\overline{x_f^*}$, here for 200 potential futures *f*, is defined. They are equidistantly distributed over the spread of the pay-offs. An equation defines for each *i* of the *n* ordered approximation points x_i the minimum of the given payout $\overline{x_f^*}$ and the endogenous approximation points x_i. These minima are termed $xx_{i,f}$[1]. They determine which linear segment on the approximation is used to evaluate the pay-off in future *f*:

$$xx_{i,f} = \min\left(\overline{x_f^*} - \overline{rp}, x_i\right) \tag{3}$$

From there, the first differences between these ordered pay-off segments $xx_{i,f}$ are defined:

$$\Delta xx_{if} = xx_{if} - xx_{i-1f} \tag{4}$$

This allows to define the approximated utility u_f^* for each future pay-off, by summing up over the first differences in the pay-offs over all segments *i*, weighted with the approximated first derivatives on each segment fd_i:

$$u_f^* = \sum_i \Delta xx_{if} fd_i \tag{5}$$

This approximation does not involve any discernable error as seem from Figure 2 above. The two extreme points $-s + rp$ and $+s + rp$ are clearly visible as

[1] Symbols with a bar such as $\overline{r}$ denote constants during an optimization approach, either the minimization of the approximation errors or later the optimization of the CPT objective in the MIP model.

they fall outside the otherwise equidistant pay-off points.

The choice of the starting points of the segments x_i and the resulting first differences between neighboring ones are optimized based on a NLP program. It minimizes the difference between u_f^* and the true value of the risk utility function at $\overline{x_f^*}$. Points close to the reference points receive a higher weight to consider the larger derivatives of the utility function around this point. Based on its solution, the true value of the utility $\overline{u_\iota}$ function at each x_i are stored. The resulting tuples $(\overline{u_\iota}, \overline{x_\iota})$ are entering as fixed and given the risk utility optimization as discussed next. The use of a NLP optimization which endogenously determines the approximation segments allows for a more accurate approximation in ranges of the function which larger derivatives.

3.2 Integration of the Piece-Wise Approximation in the Simulation Model

The integration of the piece-wise approximation is based on a set of equations drawing on so-called SOS2 (Special Ordered Sets of Type 2, GAMS, 2022: 605) variables wgt_{if}. SOS2 variables are defined over an ordered set and can take on at most two values which must be consecutive. SOS2 variables are supported by most industry MIP solvers and the usual approach to a piece-wise linear interpolation of a convex function when maximizing. The SOS2 variables ensure that the interpolation occurs only between two neighboring approximation points. Specifically, they represent here weights which define the linear interpolation on the endogenously chosen segment between two neighboring approximation points $[\overline{x_j}, \overline{x_{j-1}}]$ which encloses the endogenously simulated pay-off x_f^* in a future. Accordingly, the two selected non-zero weights $wgt_{i-1,f}$ and $wgt_{i,f}$ which refer to the starting and end point of the selected segment *i* for future *f* must add up to unity according to (6):

$$\sum_i wgt_{if} = 1 \tag{6}$$

The following Equation (7) defines the gain or loss based on the endogenously optimized pay-off x_f^* minus the reference point *rp* in each future *f*, shown on the right-hand side, as a linear combination of these endogenous weights wgt_{if} on the left-hand side. In this equation, weights are zero for all approximation points besides the ones relating to the enclosing segment according to the definition of the SOS2 variables:

$$\sum_i wgt_{if}\overline{x_\iota} = x_f^* - \overline{rp} \tag{7}$$

The approximated utility u_f^* for each optimized pay-off in any future x_f^* is then defined in Equation (8) as an interpolation, using the same weights on the linear line between the utility function values $[u(\overline{x_j}), u(\overline{x_{j-1}})]$ on the chosen segment:

$$u_f^* = \sum_i wgt_{if}\overline{u_\iota} \tag{8}$$

3.3 Endogenous Sporting of Outcomes to Consider the Weighting Function

The TK type of weighting function is used for the cumulative probabilities cp_f as shown in Equation (9):

$$w_f = \frac{cp_f^\delta}{\left[cp_f^\delta + \left(1 - cp_f^\delta\right)\right]^{1/\delta}} \tag{9}$$

These cumulative probabilities are defined from the probabilities *p* of the pay-offs x_f^* as follows for losses cp_f^- and gains cp_f^+ according to Equation (10):

$$\begin{aligned} cp_f^- &= \sum_{g\,\forall\, x_g^* \le x_f^*} p_g \\ cp_f^+ &= \sum_{g\,\forall\, x_g^* \ge x_f^*} p_g \end{aligned} \tag{10}$$

Some authors propose different δ in Equation (9) for cumulative probabilities depending on whether they represent gains or losses. This approach can be easily incorporated as the calculation of the weights attached to a cumulative probability, but not a specific future, occurs outside of the optimization model.

The weights for the cumulative probabilities are translated into subjective probabilities *sp* for each ordered future according to Equation (11):

$$sp_f = \frac{w_f - w_{f-1}}{cp_f - cp_{f-1}} p_f \tag{11}$$

Where the index *f-1* relates to the future with the next lower (losses) respectively higher (gains) cumulative probability. Equation (11) uses the differences in the subjective cumulative weightings w_f relative to the differences in the cumulate objective probabilities cp_f to define subjective probabilities sp_f for each future. In order to improve the interpretation of the outcomes, the sum of the subjective sp_f is scaled to unity.

Figure 3 shows the outcome of the weighting function, with the cumulative probabilities and the weights on the left axis and the differences in the weights on the right one, for 20 futures as used in the empirical example. The futures are defined such as to have identical objective probabilities of 5%. As seen from Figure 3, both higher losses on the left, referring to cumulative probabilities close to zero, and higher gains on the right, referring to cumulative probabilities close to unity, receive above average weights. These differences in weights, shown in red, are symmetric and depict the subjective probabilities used.

As the probabilities of the outcomes are all equally likely, any order of the pay-offs will generate the same shape of the cumulative probabilities as in shown in Figure 3, resulting in one unique fixed vector of weights. As the pay-offs for the futures and thus their ordering are unknown before optimization, the weights attached to them are also unknown beforehand. In order to attach the proper weight to a future, the position of its endogenously optimized pay-off in the ordered list of all optimized pay-offs must be known. This position determines the future's cumulative probability and its endogenous weight.

The standard approach to endogenous sorting in MIP is applied, based on an endogenous permutation matrix pm of integer variables. It maps the vector of unsorted utility values to sorted ones. This matrix carries a 1 when the unsorted utility in future f is assigned to position i in the list of sorted utility values, and zero otherwise. Note that the order of the pay-offs and of the utility values are identical, as the risk utility is increasing in the pay-off. Sorting the pay-offs hence also sorts the utility values of the futures. From an implementation perspective, sorting the utility values is preferred as this leads to the function value to optimize.

To ensure a unique mapping, the following two conditions are needed:

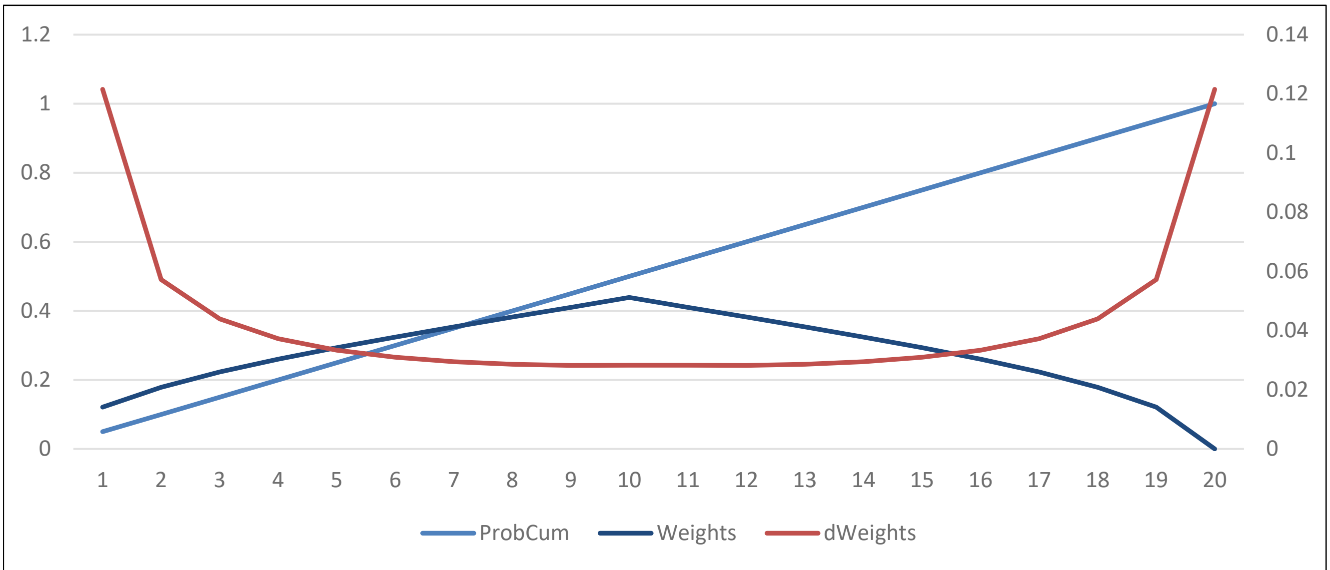
$$1 = \sum_f pm_{f,i} \tag{12}$$

$$1 = \sum_i pm_{f,i} \tag{13}$$

The approximated utility values to sort are endogenous variables. To avoid a quadratic problem where products of integer variables with continuous ones occur, the following two inequality constraints using so-called BIGM parameters are used. They ensure that empty positions in the permutation matrix pm lead to empty positions in the matrix of endogenous permutation values vm :

$$vm_{f,i} \leq pm_{f,i} \ BIGM^+ \tag{14}$$

Figure 3. TK weighting function for 20 equally likely futures



Source: Authors

$$vm_{f,i} \geq pm_{f,i} \; BIGM^{-} \quad (15)$$

Where $BIGM^{+}$ is the utility attached to the maximum utility considered in the approximation and $BIGM^{-}$ the minimal one.

The following two related constraints ensure that the value in the matrix *vm* is equal to the approximated utility where the permutation matrix *pm* shows the indicator value 1, and zero otherwise:

$$u_f^* \leq vm_{f,i} + BIGM^{+} \left(1 - pm_{f,i}\right) \quad (16)$$

$$u_f^* \geq vm_{f,i} + BIGM^{-} \left(1 - pm_{f,i}\right) \quad (17)$$

The sorted outcome us_i^* of u_f^* are than defined as:

$$us_i^* = \sum_f vm_{f,i} \quad (18)$$

The objective function maximizes the weighted sum of the sorted approximated risk utility us_i^*:

$$u = \sum_i \overline{sp}_i \, us_i^* \quad (19)$$

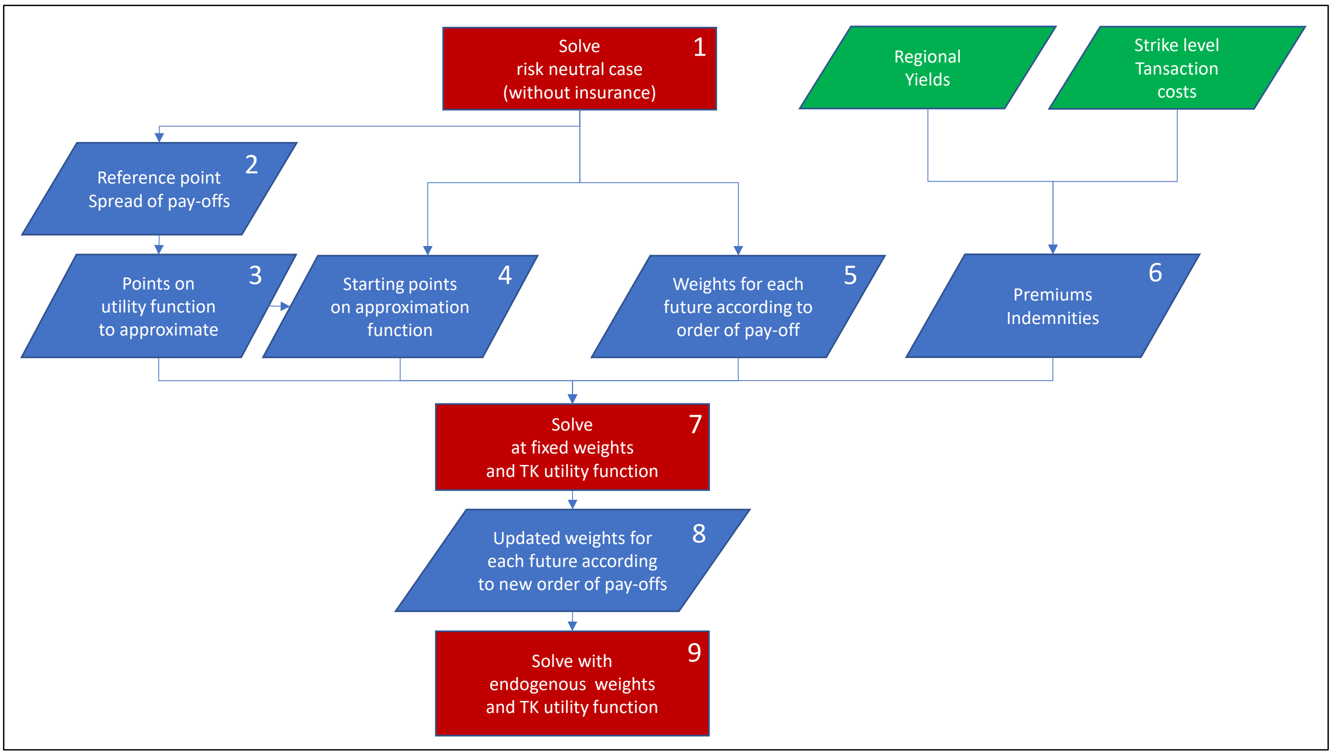
3.4 Solution Strategy

In order to find the approximation, the model is first optimized under risk neutrality (step 1 in Figure 4), without the additional variables and equations required for CPT. The resulting optimal risk neutral pay-offs define the reference point and the considered spread of the pay-offs (step 2), both needed to find approximation points for the utility function (step 3).

For each optimal pay-off in each future under risk neutrality, the enclosing segment on the approximation function is next searched, and weights $wgt_{i-1,f}$ and $wgt_{i,f}$ calculated with recover exactly this pay-off. From there, the attached approximated utility outcomes u_f^* are calculated (step 4). These outcomes are subsequently sorted to determine starting values for $vm_{f,i}$ and $pm_{f,i}$ (step 5). This allows to calculate also starting values for the sorted outcomes us_i^* and the objective variable u. Jointly, this defines an integer feasible (but most probably not yet optimal) solution to the CPT problem, at the previously simulated optimal solution under risk neutrality.

The data on regional yields and assumptions on the strike level and transaction costs allow to define the crop specific premiums and indemnities (step 6). These are attached to variables which allow to insure each crop up to its endogenously determined acreage. These variables are added to the optimization problem.

Figure 4. Overview on modelling process



Source: Authors

From the integer optimal starting point under risk neutrality, a new optimum under CPT is searched for (step 7), considering the new options to insure, but with a still fixed permutation matrix pm. Optimizing this problem from the given integer feasible point solves typically in a few seconds. Besides opting into insurance, also the farm program might change. The resulting solution is probably inconsistent as the optimal pay-offs are not yet endogenously sorted; their order is unchanged from the risk neutral solution. Accordingly, the updated outcomes u_f^* are sorted again and new starting values derived for $vm_{f,i}$,$pm_{f,i}$ as well us_i^* (step 8). This results in a second integer feasible starting point, with potentially updated weights for the futures. After releasing the bounds on pm, the final full problem is solved where also the ordering and thus the weights attached to each future become endogenous (step 9). This solve is numerically far more demanding and can take several minutes[2].

4 An Empirical Example Application

4.1 Simulation Model

The bio-economic farm-scale model FarmDyn[3] is employed in the empirical application, taking the uptake of crop insurance as an example to evaluate the implementation of the approximated CPT approach. FarmDyn is quite evolved, considering, for instance, indivisibilities in investments into a larger set of different machines, bi-weekly farm labor constraints and details of the restrictions under the so-called "Greening" of the Common Agricultural Policy (HEINRICHS et al., 2021a) or the German Fertilizer directive (KUHN et al., 2020). It is based on MIP, realized in GAMS (General Algebraic Modelling Language, GAMS, 2022, an Algebraic Modeling Language widely used in agricultural economics, see BRITZ and KALLRATH, 2012) and steered by a Graphical User Interface[4] based on GGIG (BRITZ, 2014). Using a model with detailed decision variables and constraints allows to assess the numerical burden of the proposed CPT implementation in an empirically relevant environment.

The parameterization of FarmDyn draws to a large extent on the highly detailed farm-management planning data offered by KTBL, a German parastatal, which consider, for instance, effects of plot-size and plot-farm distance on costs, machinery requirements and labor needs of individual farm operations (HEINRICHS et al., 2020b). KTBL also reports matching time series of regional crop yields. FarmDyn is based on a modular concept (BRITZ et al., 2021); the implementation of the CPT approach is one example for such a modular extension. FarmDyn can depict different farm branches in detail. Here, solely the module for arable cropping is used, restricting the choice of technologies to plough based tillage under conventional farming.

4.2 Case Study Farms and Stochastics

An arable farm situated in the region around Cologne in Germany with 100 hectares serves as the study case for the empirical application. It can cultivate winter wheat, winter rape, summer barley or summer peas. It is assumed that harvesting is outsourced to a contractor, all other field operations are managed by the farm itself based on own machinery. Related investment costs are treated in the comparative-static set-up as if they were variable costs, no sunk costs are assumed. It is also assumed that the farm owns the land, which can be important when considering risk behavior due to wealth effects.

The farm is subject to the Ecological Focus Area (EFA) requirements of the Common Agricultural Policy such that it has to dedicate 5% of its cropped land to specific types of land cover. The considered options comprise idling land, catch crops in combination with summer crops or cropping summer peas as a legumi-

2 As usual with MIP problems, the problem is not solved to full optimality. The relative optimality tolerance is set here to 0.2% and an absolute one of 10, the latter refers to farm household income in € for the risk neutral case and the risk utility level under CPT. Usually, the solver tends to find quite fast good integer optimal solutions under CPT, but requires many iterations to move the best bounds close enough to them.

3 The FarmDyn documentation can be found at https://farmdyn.github.io/documentation/. FarmDyn is open source, available at https://svn1.agp.uni-bonn.de/svn/dairydyn/trunk with userid *farmdyn* and password *farmdyn*. The code of the equations discussed above can be found in *gams/model/stochprog_module.gms*, the approximation of the utility function and solving the CPT version in *gams/solve/run_cpt.gms* and the calculation of the indemnities and premiums in *gams/coeffgen/stochProg.gms*.

4 The user interface allows, for instance, to input the parameters for the TK utility function and the probability weighting, the number of approximation points to consider and to select data with regional yields.

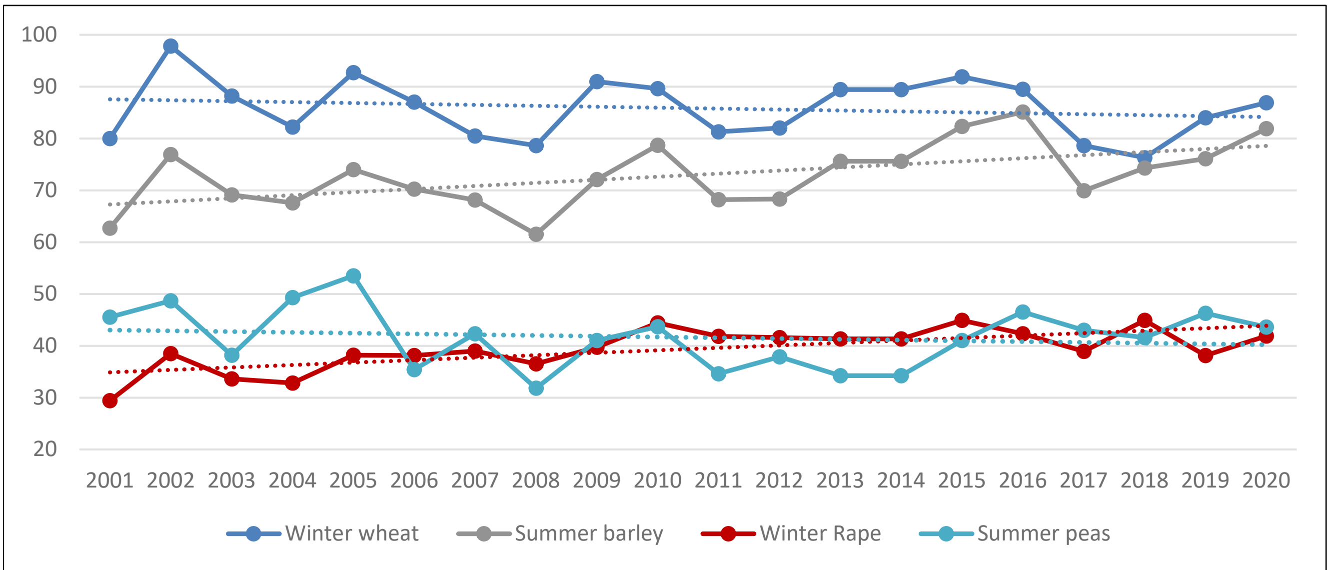
nous crop, where catch crops only count with a factor of 0.3. Furthermore, the shares of the two most important crops cannot exceed 95% and the one of the most important crop not 75% of the acreage under the Common Agricultural Policy. The crop choice is further restricted by crop and crop group specific maximum crop shares. In the comparative-static set-up, these maximum shares capture the effect of minimum waiting time requirements before a crop or group of crops can be cultivated on the same plot again. Crop yields, prices, costs, labor and machinery requirements are taken from the KTBL data base. The farmer can work up to 1,700 hours per year, and each month up to 1/4 above an equal distribution of the 1,700 hours over the year. The model considers returns-to-scale for labor needs related to managing the farm and the arable cropping branch. It is also possible to partly work off-farm. Accordingly, the objective relates to farm-household income. Due to this set-up, risk management strategies are considered even without crop insurance. Crop shares can be adjusted to benefit from not fully correlated crop yield variations, and labor can be shifted to off-farm use where risk-free returns in form of deterministic wages are assumed.

Given the focus on crop yield insurance, crop yields are treated as stochastic, using observed yields for the Regierungsbezirk Köln for the years 2001-2020 (see Figure 5). Yield levels and the size of the fluctuations underline that the region benefits from better soils and a climate quite suitable for arable cropping under rainfed conditions. Expected crop prices are set to the three-average 2018-2020. The yield series were detrended based on linear trends (dotted lines), trend estimates for the last observed year 2020 define the expected yields. The errors from the trend lines are added to the expected yields to define the yield risk faced by the farmer. The graphs already highlight that the downside risk differs across the crops, and no catastrophic events were observed. The minimum relative yield level for winter wheat, the dominant crop from an economic view point, is around 91%, opposed to around 75% for summer peas and 87% for winter rape and summer barley.

As the insurance product is hypothetical, different strike levels are considered in sensitivity analysis, between 75% and 100% in steps of 5%. BABCOCK (2015) also considers different strike levels, as famers can decide on them when opting into subsidized crop insurance in the US. Equally, besides a base cost of 10 € per ha, three levels of transaction costs on paid-out indemnities are considered, at 20%, 40% and 60%. This could be interpreted as analyzing potential levels of subsidization. The scenario which is discussed in some detail is based on a 100% strike level combined with 20% transaction costs; it represents the case where the highest share of the land is insured under the considered options.

Different from DALHAUS et al. (2020), no yield time series for individual farms are available. Accordingly, the basis risk cannot be quantified which likely implies an overestimation of the benefits from crop insurance. But the example still allows to evaluate the proposed implementation of CPT in an empirical setting from a computational viewpoint as well as generating interesting insights from optimizing insured areas and crop shares simultaneously.

Figure 5. Observed yields 2001-2020, in dt/ha



Source: KTBL data base. Dotted lines show linear trends.

5 Results

5.1 Risk Neutral Solution

To gain insights in the optimization logic, results for the risk neutral case are discussed first. They define the reference point. The farmer devotes around 51 hectares to winter wheat, 29 hectares to summer barley and 15 hectares to winter rape. A combination of 3,3 ha idling land, 2,2 ha of summer peas and 1,4 ha of mustard as a cover crop fulfill the EFA restriction. About 800 hours are allocated to field operations and 540 hours for general farm and crop branch management. This allows the farming household to work around 340 hours per year off-farm at minimum wage, under an option where no social security or income tax paid. In July, August and September, the maximal allowed deviations (+25%) from the assumed mean distribution of the labor hours over the year become binding and no leisure time is possible. In these months, around 180 hours would be worked, in contrast to solely around 100 hours in January and December. The endogenous labor distribution also considers to take holidays when working off-farm and to unevenly distribute the work hours needed for farm management, with a flexibility of 50%.

The resulting expected farm-household income amounts to around 92.600 €, of which 30.000 € stem from first pillar payments and are risk-free. The same holds for around 3.000 € of wage income. Yearly fluctuations due to production risk imply a range of the realized incomes between around 75.000 € and 114.000 € (see also Table 1 below). The average arable crop farm in the federal state of NRW has a somewhat higher income of around 117.000 € at 97 ha in the reporting year 2020/2021 (BMEL, 2022), but as a statistical average shows some animal production and 10 ha of fodder production and uses considerably more labor. It also crops potatoes and sugar beet, not considered here due to the specific investment requirements in machinery and buildings. Against this statistical average, the chosen farm size and the simulated farm income seems reasonable.

5.2 CPT

The expected mean income from the risk-neutral case serves as the reference point for the follow-up CPT based optimization. Under CPT, futures with losses now receive higher weights, due to a larger negative slope for losses on the risk utility function and to subjective probability weighting.

Table 1. Key simulation metric for the 20 futures

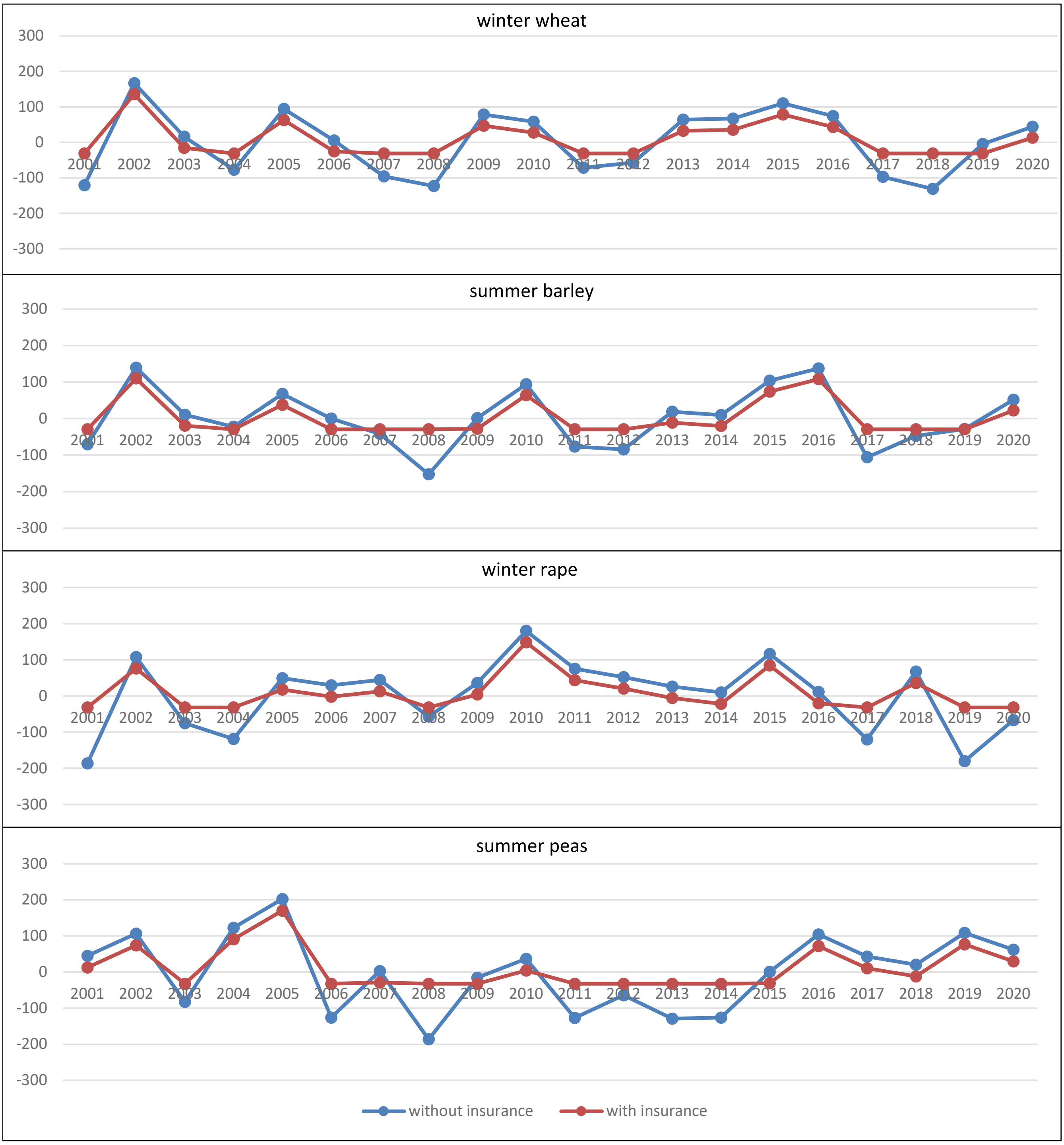
	Objective Probality	Subjective Probability	Pay-off Neutral	Risk Utility Neutral	Pay-off CPT	Risk Utility CPT	Approx. Risk utility CPT	Approx. Error absolute	Approx. Error %
2001	**0,050**	**0,070**	**76.864**	**-10.935**	**81.144**	**-8.264**	**-8.259**	**-5,78**	**0,07**
2002	0,050	0,140	114.310	6.559	109.587	5.287	5.282	5,75	0,11
2003	0,050	0,030	92.974	198	88.946	-3.007	-2.965	-41,96	1,40
2004	0,050	0,040	83.826	-6.530	84.021	-6.402	-6.361	-40,88	0,64
2005	0,050	0,040	104.417	3.845	99.476	2.392	2.371	20,83	0,87
2006	0,050	0,030	93.160	275	88.934	-3.016	-2.974	-41,68	1,38
2007	0,050	0,040	83.662	-6.638	85.162	-5.643	-5.599	-44,08	0,78
2008	**0,050**	**0,140**	**74.281**	**-12.503**	**81.144**	**-8.264**	**-8.259**	**-5,78**	**0,07**
2009	0,050	0,040	99.378	2.362	95.017	961	933	27,12	2,82
2010	0,050	0,040	104.499	3.868	99.825	2.498	2.472	25,85	1,03
2011	0,050	0,030	84.139	-6.324	85.772	-5.232	-5.192	-40,12	0,77
2012	0,050	0,030	84.691	-5.958	85.708	-5.276	-5.235	-40,75	0,77
2013	0,050	0,030	98.654	2.139	94.491	777	758	18,97	2,44
2014	0,050	0,030	98.211	2.002	94.063	622	614	7,83	1,26
2015	0,050	0,070	107.971	4.843	103.448	3.567	3.520	46,64	1,31
2016	0,050	0,050	105.346	4.109	100.460	2.689	2.656	33,42	1,24
2017	**0,050**	**0,050**	**78.082**	**-10.185**	**81.989**	**-7.724**	**-7.717**	**-6,26**	**0,08**
2018	0,050	0,040	80.981	-8.368	84.381	-6.164	-6.120	-43,28	0,70
2019	0,050	0,030	88.279	-3.489	85.360	-5.510	-5.467	-43,26	0,79
2020	0,050	0,030	97.614	1.814	93.200	292	268	23,48	8,04
Max			114.310	6.559	109.587	5.287	5.282	46,64	8,04
Min			74.281	-12.503	81.144	-8.264	-8.259	-44,08	0,07
Mean			92.567	-1.946	91.106	-2.271	-2.264	-7,20	1,33

Note: Approx.: approximation based on linear interpolation
Source: Model simulation results by authors

To explain choices on insured area and crop share adjustment, we discuss one case in some detail. It refers to a 100% strike level and 20% transaction costs where the maximal acreage is insured. The premiums of the insurance amount to around 30-32 € ha^{-1}, depending on the crop. This can be compared to the around 10 € ha^{-1} as the cost of hail insurance according to the planning data used which are already considered in the risk neutral case. 84 ha are insured, comprising all the winter wheat and summer barley acreage, plus around 4 ha of rape equivalent to about 30% of its area. Considering yearly premiums to pay and deducting the indemnities paid out, resulting expected yearly costs of the insurance amount to around 1.450 € at farm level or around 15 € ha^{-1}.

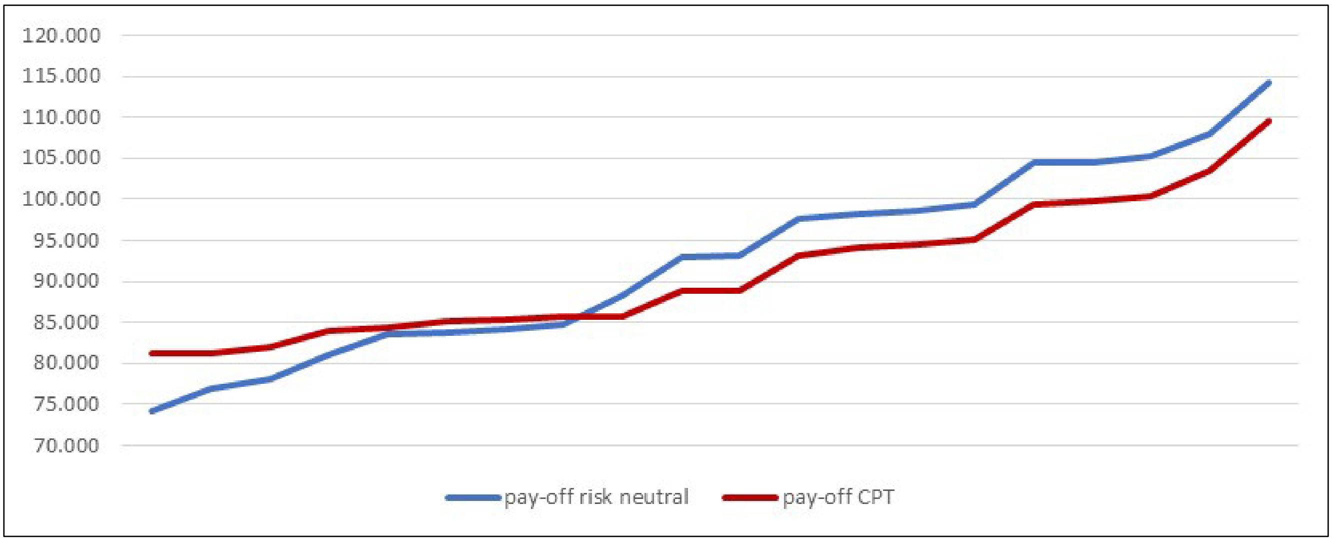
Besides opting into insurance, the crop choice is adjusted under CPT as well: the winter wheat area is increased by one hectare and summer barley reduced

Figure 6. Yearly fluctuation in revenues per ha, with and without insurance



Source: Authors' calculations based on KTBL data, indemnities and insurance costs at 95% strike level and 20% transaction costs on indemnities paid plus 10 € per ha.

Figure 7. Order pay-offs under risk-neutrality and CPT



Source: Model simulation results by authors

accordingly. Moreover, the 2,2 ha summer peas found under risk neutrality are dropped from the cropping program and the idling area is increased by 2 ha instead to fulfill the EFA requirement.

A look at the absolute fluctuations of revenues per hectare (Figure 6) for the insured and not-insured case helps to understand this choice. Wheat as the dominant crops which the largest contribution to farm revenues shows damages in many years (2001, 2004, 2007, 2008, 2011, 2012, 2017, 2018)[5]. The deviations of the summer cereal as the second most important crop typically show below average yields in these years as well; they tend to be somewhat larger.

In contrast, winter rape sometime shows positive fluctuations (e.g. 2011, 2012, 2018) or limited negative ones, only (2008), where damages in cereals occur, or larger damages where negative deviations in cereals are limited (e.g. 2001, 2019). Cropping rape together with cereals therefore dampens their joint production risk and reduces the incentive to insure the winter rape area if cereals are already insured. Accordingly, the farmer insures the two major crops fully and complements this choice by insuring part of the winter rape, only.

Details on the pay-offs and related utility values can be found in Table 1. The total risk premium as the difference between the new optimal income of around 91.100 € and the risk neutral one of 92.600 € amounts to around 1.500 €. It is almost equal to the differences between the insurance premiums paid and the indemnities received as the impact of the adjustment of the crop choice on the expected pay-off is quite limited.

In Table 1 above, the three years with the highest losses under CPT are shown in bold, they receive also larger subjective weights, as do the cases with the largest gains. The largest indemnities are paid out in the three years with the extreme losses, in 2008 with around 10.900 €, a year with the same income as in 2001, where around 9.000 € of indemnities are paid, and 2017 with 8.500 €. The table shows also that relative approximation errors are quite small. The largest on is found in the year 2020 with 8%. This case simulates a pay-off very close to the reference point where the value on the utility function is close to zero such that even smaller absolute deviations lead to a large relative error.

The differences in the ordered pay-offs are visualized in Figure 7 above. As expected, the CPT solution turns the pay-off curve clockwise and reduces the risk in years with larger losses. This comes at the expense of lower payouts in 12 years, such that the expected pay-offs under CPT are higher in 8 out of 20 years. Despite the focus of CPT on the extremes, where also larger gains carry high weights, the maximum income observed under risk neutrality is reduced by around 5.300 € when moving to CPT. Of this, around 2.600 € are premiums paid. The rest stems from adjustments in the farm program, especially from dropping 2,2 ha of summer peas and increasing the idling land instead.

5 Observed realized yield fluctuations in ex-post years define here distinct futures or state-of-nature from the viewpoint of the optimizing agent. For simplicity, these futures are here named after the ex-post years.

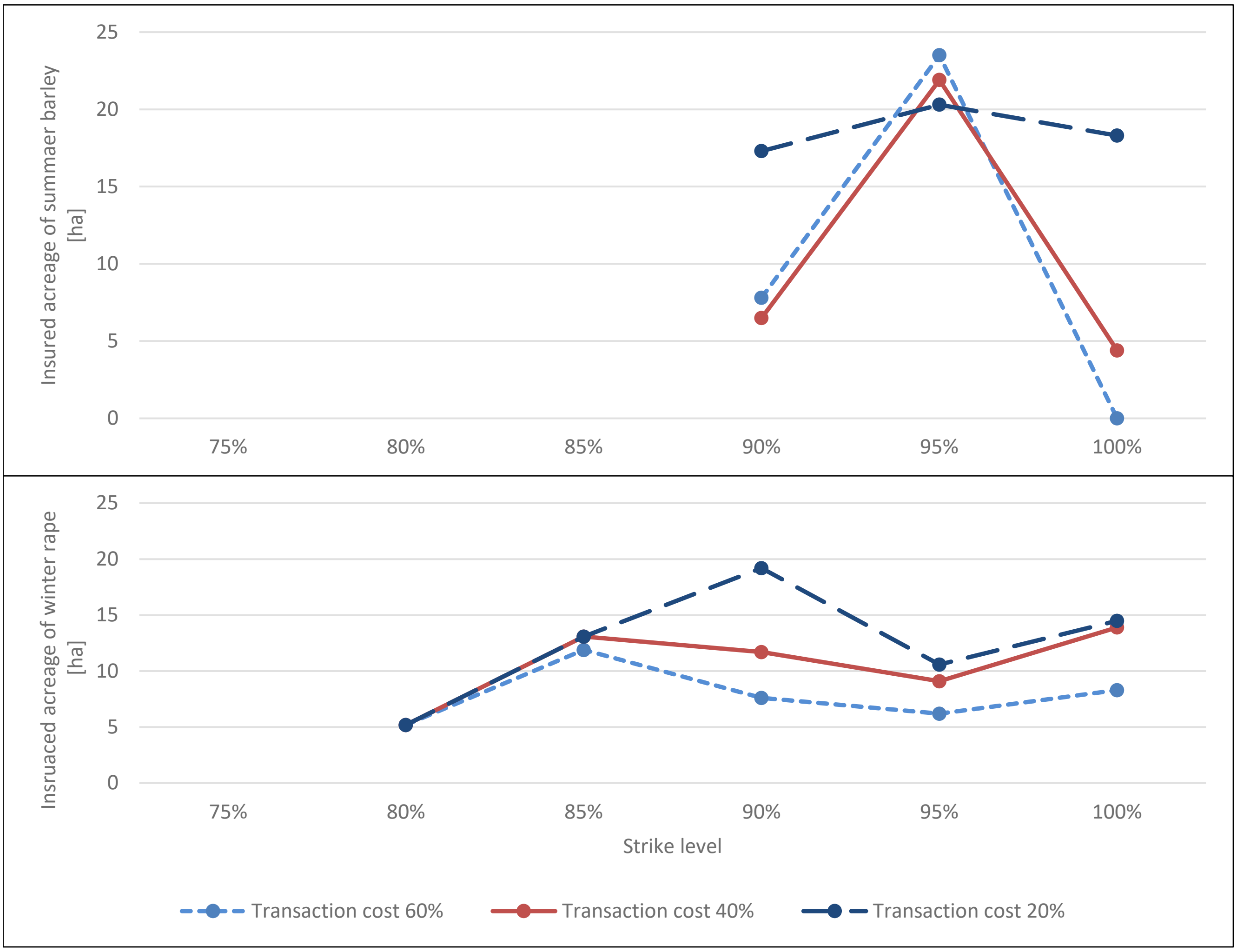
5.3 Sensitivity Analysis

In order to enrich the analysis and to test the approach further, results for the other considered parameter values of the crop insurance are discussed. Figure 8 depicts two main results, the insured acreage and resulting costs at farm level, i.e. premiums minus indemnities. It highlights that lower transaction costs at unchanged strike level let the insured area increase or at least keep it unchanged. Especially at high strike level, the insured acreage reacts quite sensitive to the cost of the insurance. At a 95% and 100% strike level, close to all cropped area is insured under low transaction cost (20%) or no area at all under high transaction cost (60%). In parallel, the cost of insurance at farm level tend to increase or at least stay unchanged if transactions costs are reduced, such that the expansion effects in insured acreage dominates with regard to total costs. In opposite to a contract variant analyzed by DALHAUS et al. (2020), the farmer pays premiums in our set-up also in years where indemnities are paid out. Higher transaction costs hence decrease the impact of insurance on downside risk reduction, even if all insured crops would show damages in a year.

As seen from the spread of the insured costs in Figure 8, these adjustments hardly matter from an economic perspective. On per hectare basis, insurance costs fluctuate between close to zero at the 80% strike level and a maximum of around 15 €, found under a 100% strike level and 20% transaction costs.

The choice of which crops to insure to which extent reacts quite sensitive to the offered insurance variant (see Table 2). Winter wheat with its minimum observed yield above 90% of its mean can be insured only at quite high strike levels of 95% or 100%. This option then implies also larger costs and is chosen solely at low transaction costs of 20%. This holds also for partial coverage of the winter rape area. In opposite, summer barley areas are partly insured also at higher transaction cost levels. At low strike levels, summer peas are insured, only, and the related insurance costs negligible at farm level due to the low insured acreage overall.

Figure 8. Insured acreage and resulting insurance costs at farm scale



Source: Model simulation results by authors

Table 2. Insured areas by crop (ha) under different strike levels and transaction costs

	Transaction cost	Strike level 75%	80%	85%	90%	95%	100%
Winter wheat	60%						
	40%						
	20%					51,7	51,8
Summer barley	60%				12,7		
	40%				15,9	20,8	16,5
	20%				22,0	28,3	28,2
Winter rape	60%						
	40%						
	20%				7,0	2,7	4,2
Summer peas	60%						
	40%	1,0	1,0				
	20%	1,7	1,7				

Source: Model simulation results by authors

Crop acreages of the three major crops react moderately to the offered insurance variant (see Table 3). The total amount of cereals is often bounded by a maximum of 80% of cereals allowed on total cropped land. After accounting for the EFA requirements, the remaining area is then used for winter rape. Summer barley shows acreages between 26 and 28 ha depending on the offered insurance variant, the spread for winter rape and winter wheat is smaller which around 1 ha. More volatile are land uses linked to the EFA requirement, especially in relative terms. The idling land covers between 3,3 and 5 ha, it is either combined with summer peas (between 0,3 and 1,7 ha) and/or with cover crops (between 0,9 and 1,6 ha). Besides overall land available, the legal obligation to fulfill the EFA requirement and the maximal cropping share of cereals, labor availability determines to a large extent the chosen cropping program. In summer and late autumn, the maximally assumed work load is reached. This explains why idling land is found as part of the crop choice. Cropping summer peas instead to fulfill EFA requirement would generate additional farm income, but would require additional labor in peak periods and thus reduces the amount of off-farm labor and related wage income in each and every month. Combined with some larger down-side risk, summer peas therefore react quite sensitive to changes in the offered insurance.

Table 3. Acreages (ha) under different strike levels and transaction costs

	Transaction cost	Strike level 75%	80%	85%	90%	95%	100%
Winter wheat	60%	52,6	52,6	52,6	51,7	52,6	52,6
	40%	52,0	52,0	52,6	50,9	52,1	52,6
	20%	51,6	51,6	52,6	51,5	51,7	51,8
Summer barley	60%	25,9	25,9	25,9	28,1	25,9	25,9
	40%	26,5	26,5	25,9	29,1	27,2	25,9
	20%	27,0	27,0	25,9	28,5	28,3	28,2
Winter rape	60%	16,5	16,5	16,5	15,6	16,5	16,5
	40%	16,5	16,5	16,5	15,4	15,9	16,5
	20%	16,4	16,4	16,5	15,5	15,5	15,5
Summer peas	60%	0,0	0,0		0,3	0,0	0,0
	40%	1,0	1,0				
	20%	1,7	1,7				
Mustard as cover crop	60%		0,0	0,0	1,4	0,0	0,0
	40%		0,0	0,0	1,5	0,9	0,0
	20%		0,0	0,0	1,6	1,6	1,6
Idling land	60%	5,0	5,0	5,0	4,3	5,0	5,0
	40%	4,0	4,0	5,0	2,9	4,7	5,0
	20%	3,3	3,3	5,0	4,1	4,4	4,5

Source: Model simulation results by authors

5.4 Some Technical Aspects

The implementation of the approximated risk utility function requires additional equations and variables, including the SOS2 variables to select the relevant segments on the approximated utility function and the integer variables for sorting the outcomes. This provokes additional computational burden. Indeed, while the solution of the MIP model under risk-neutrality requires just 2 seconds[6], the CPT case with the SOS2[7] variables for each future drive up the solution time to around 10 seconds as long as permutation matrix is fixed. If the endogenous sorting is activated for the full solve, many integer variables are added and the solution increases to up to around five minutes.

These run times reflect some specific choices to speed up solution. After solving the risk neutral case, bounds of 8.000 € around the risk-neutral pay-offs in each future and around the sorted outcomes were introduced, from which also bounds on the related utility function values were derived. The size of bounds is chosen such they not become binding during optimization; they hence do not impact the optimal solution. Segments on the interpolation function for a future outside the related bounds were excluded from the solution, equally, the permutation matrix was fixed to zero for combinations excluded by these bounds. This reduces the solution space which matters especially for the integer variables. Without these bounds and fixed integers, the solve process with endogenous sorting was in some of the sensitivity experiments stopped by a maximal time limit of one hour, compared to maximal five minutes with these bounds. Moreover, FarmDyn already comprise heuristics which remove endogenous variables not found in a relaxed integer solution, solved before the risk neutral case is optimized. This underlines that besides the implementation of the equations discussed above, further modifications to existing model code might be necessary to deal with the increased computational burden provoked by the CPT implementation.

Due to the subtraction of the reference point and the power utility function, the CPT approach tends to generate objective values which are of at least a magnitude lower than the average pay-offs under risk neutrality. This must be kept in mind when setting the relative or absolute optimality tolerance of the MIP solver different from zero.

6 Discussion

6.1 Methodological Viewpoints

The empirical application underlines that the proposed approximation of the TK utility function works almost perfect and remaining errors are negligible from an empirical perspective. The approximation can be applied to any risk utility function, independent from prospect theory. If this risk utility function is concave, the SOS2 variables can be replaced by fractionals. The endogenous subjective probability weighting is necessary for a CPT implementation, only. In the proposed form, it is applicable only if all considered futures are equally likely. In case where futures are constructed based on random draws from distributions, the scenario design must be adjusted accordingly.

Analyzing the uptake *rate* of an additional risk-management instrument, such as crop insurance in our empirical example, renders is impossible without actual optimization to determine the distribution of the pay-offs and their order ex-ante, even if the farm program would be otherwise fixed. This is different in other literature where the decision is analyzed whether or not to insure all crop area at a given farm program. In this case, the risk utility of the two considered options (with and without insurance) can be calculated and compared under CPT. Applying the logic underlying the optimization approaches proposed by HENS and MAYER (2014) and LEVY and LEVY (2004) instead of what is proposed here to constrained optimization is challenging. These approaches require first to construct efficient M-V points to choose from. In a constrained optimization model, this would mean to implement an approach such a MOTAD and solve it at different levels of risk aversion, before searching the best one under the CPT criterion over the set of optimized M-V solutions. Crops yield distribution are typically skewed and can therefore not be normally distributed (cf. CONRADT et al., 2015). This violates the condition by LEVY and LEVY (2004) which prove formally that normality (besides other conditions) is required to render the CPT efficient solutions a subset of the M-V frontier. Picking the best solution under CPT from solutions optimized under MOTAD or M-V and different levels of risk aversion is therefore not guaranteed to find an (approximate) overall best CPT

6 The tests were run on quite performant computer with a i9 processor with 8 cores and 32 GB of memory, using GAMS 39.2.1 and GUROBI as the MIP solver.

7 After some experimenting with different solvers, solver options and further technical details of the code implementation, the SOS2 mechanism is implemented in the code based on SOS1 variables as this sped up the solution time. Details are not discussed further in here.

solution. Additionally, generating many M-V optimal solutions can be computationally intensive if the model is a MIP. Moreover, it requires additional code to design, run and collect these experiments.

The proposed implementation is based on distinct futures. This is relatively common in optimization approaches and also proposed in existing approaches to maximize utility under CPT by HENS and MEYER (2014), LEVY and LEVY (2004) and COELHO et al. (2012). It gives full flexibility with regard to the underlying data generation process. If not observed time series of stochastic variables are used as here, but random draws, deciding on a set of representative outcomes can be challenging. Approaches such as Latin Hypercube Sampling considering co-variances (IMAN and CONOVER, 1980) or scenario reduction (see for instance SPIEGEL et al., 2020) can help to escape the curse-of-dimensionality in this case.

A challenge provides, however, the combination of distinct futures and the subjective weighting. The marginal utility of a change in the pay-off in a specific future under CPT depends on the marginal change on the power utility function times the weight of this future. Under the TK weighting function, the differences in weights solely depend on the order of the pay-offs, not on their differences. Large differences in the weights and thus in the marginal utility between the ordered pay-offs can, therefore, be accompanied with quite small differences in their pay-offs. This is found in our example (see Table 1). The optimal pay-offs in the three cases with the most extreme losses differ by less 900 €, which is around 3% of the spread of the pay-offs and one percent of their mean. But their weights differ between 5% and 14%. Improving the pay-off in the worst case carries hence a very large weight, even compared to other cases with similarly large losses. Using distinct futures, this leads also to order dependent jumpiness in the marginal utility, a reason why the optimization with endogenous ordering is computationally demanding. This explains the perhaps at first glance curious result that the two smallest simulated pay-offs are exactly equal (see Table 1). If the pay-off of the worst future would be improved at this point by a change of the farm program or of insured acreages, their order would be likely reversed. The resulting switch of their weights would likely trigger a re-adjustment in the opposite direction.

The implementation of CPT or other risk-utility functions in a constrained optimization model requires parameters which describe in quite some detail the risk behavior of the agent. While there is little doubt that the general insights into risk behavior from experiments are valid, to which extent actual parameter values from experiments can be transferred to the field has been debated for long (cf. LEVITT and LIST, 2007). One way to advance here is to check the sensitivity of simulated outcomes to the parameter choice, similar to the HARDAKER et al. (2004) concept of stochastic dominance analysis with respect to a function. DALHAUS et al. (2020) therefore analyze different parameter sets from literature. Another uncertainty related to CPT addresses BABCOCK (2015) by considering different farm income indicators to define gains and losses and risk utility.

Instead of exploring new types of risk utility in constrained optimization which require (agent specific) parameters relating to risk utility, SPIEGEL et al. (2021) propose to optimize the farm program under constraints depicting second-order stochastic dominance against a given benchmark. Such an optimized program should be preferred by any risk-neutral or risk-averse decision taker, but not necessarily by an agent under CPT which implies risk-loving on the loss segment. DALHAUS et al. (2020) therefore define crop yield insurance contracts which are attractive to farmers under EU and CPT.

Another approach in the context of CPT and constraint optimization is taken by HUBER et al. (2020). They apply CPT to assess the satisfaction level of the farmer with their current farming program. They compare the prospect value of a number of past realizations of farm income with a reference income level in a recursive-dynamic setting. If this prospect value undercuts the reference level, new farming options are searched for, in their example with regard to weed control, and a new farm program is optimized in a deterministic setting to maximize farm income.

6.2 Computational Considerations

The use of the SOS2 variables and the integers required for the endogenous sorting increases the computational burden compared to a standard risk-free model considerably. Modern MIP solver have efficient algorithm to implement the required SOS2 variables. The number of the related integers for the approximation of the utility function is determined by the product of the approximation segments and futures. Tests not documented here showed that using fewer approximation segments slightly speeds up the solution process, but increases as expected the maximal errors in the approximation. It had, however, here no discernable impact on the simulated crop acreages and insured areas. But the low sensitivity found in this respect might depend on the specific case.

The number of integer variables related to the endogenous sorting of the pay-offs is equal to the square of the number of considered futures, and seems to provoke a higher computational burden compared to SOS2 variables linked to the approximation of the utility function. If observed time series are used and the solution time is considered critical, the user could only drop some observations to reduce the number of considered futures to speed up the solution. This could be solely recommended if far more observations than for two decades such as in our example would be available. If random draws from distributions are used, it might pay off to check carefully how many futures are necessary for an accurate representation of risk to avoid unnecessary long solution times.

For optimization models so far not using integers, implementing the discussed CPT solution likely requires a change in the solver. Its implementation into constrained optimization models with non-linear constraints which are not quadratic might prove especially challenging. The set of available general mixed non-linear solvers required in this case is still limited and the computational burden to solve this type of model is found as quite high (cf. VIGERSKE, 2017).

An alternative to the computationally demanding endogenous sorting is the use of iterative updates to then fixed weights for the futures. Tests have shown that this approach can provoke cycling. More importantly, it is impossible to avoid solutions under fixed weights which propose extreme losses in futures with lower subjective weights as the optimum. This is inconsistent with the assumptions of CPT.

6.3 Empirical Example and Modelling Approach

The analyzed crop insurance product does not exist in the region, it is hypothetical. Therefore, a rather simple and transparent option is chosen which bases the insured risks on reported regional yields. Such insurance products are available for instance in the US since decades as so-called Area Yield Protection plans (cf. SKEES et al. 1997). They carry a basis risk as farm specific yield fluctuations differ from regional ones (see FINGER, 2012). Basis risk cannot be assessed here due to missing matching time series of farm-scale yields. In order to concentrate on the CPT implementation, no evolved crop yield insurance options such as index-based products are here designed and evaluated under CPT, such as by DALHAUS et al. (2020). This would probably also be of limited merit if basic risk cannot be considered.

Equally, no state-contingent crop management is considered. This assumption might be challenged. For instance, drought damage early in the growing season under rain-fed conditions might trigger down-side adjustments in costly plant-protection measures which reduces the financial risk related to volatile yields. On the other hand, the damage related to crop failures in here solely considers the revenue loss. In reality, further costs might occur, such as interest on additional loans needed to cover production costs if revenues are no longer sufficient to do so.

The analyzed volatility relates to a single year. Leaving discussions about climate change aside, there is no obvious reason to assume that weather events in follow-up years are strongly positively correlated, such as having with a higher probability a dry after a dry or a wet after a wet year. This means that summed up weather driven yield fluctuations over multiple years are likely to cancel each other out, at least partially (cf. ODENING and SHEN, 2014). Furthermore, if farmers have some financial buffering capacity, for instance, by credit lines on their current account or the possibility to postpone investments at farm or household level, additional risk management options exist which are not considered here. Such options further flatten the distribution of farm incomes over multiple years. SPIEGEL et al. (2020) therefore consider the distribution of wealth defined as the Net-Presence Value of the farm operation over a longer simulation horizon when assessing production and market risks, instead of focusing on inter-annual fluctuations. It is likely that agents treat risk in final wealth different from risk in yearly income. Accordingly, the parameter choice in the risk utility function needs to consider the simulated time horizon. Equally, if parameters are taken from experiments, the time horizon of the simulation model should match the framing of experiments.

The detailed discussion of one of the analyzed insurance option underlines that the optimization approach fine-tunes the risk management options to the considered futures (or state-of-natures). This can render the results, at least in detail, quite sensitive to specific realizations of random variables. It is, therefore, not unlikely that, for instance, prolonging the yield time series once new data become available could provoke some changes to the optimum solution. Changes are especially likely if new observations change the extreme cases which carry a high weight in overall risk utility. Classical moment-based approaches such as E-V are less sensitive in this respect.

Finally, the reader is reminded of some obvious consequences of offering (subsidized) crop insurance, also shown by our results. Opting into crop insurance, as long as it is not fully subsidized, can make farmers poorer, only, even if subjectively better off, if it does not substitute some more expensive pre-existing risk management strategy. This asks for a careful evaluation of the usefulness of subsidized crop insurance, especially in cases where the production risk is moderate and unlikely to threaten the survival of the farm.

7 Summary and Conclusions

The proposed combination of endogenous sorting of the pay-offs and a piece-wise linear approximation of the risk utility function allows to optimize risk utility under CPT in MIP based simulation approaches. In its current form, it is applicable only if all futures are considered equally likely. The implementation opens the door for the combination of experiments and econometrics to estimate parameters related to CPT and follow-up optimization in evolved farm-scale models to simulate the optimal portfolio of existing or novel risk management instruments, and quantify from there economic, social and environmental indicators. Such analysis is especially rich if parameters related to risk utility are farmer and farm specific and larger samples of farms are analyzed.

The CPT implementation is explored here by analyzing the uptake of crop insurance in a case-study farm in Germany, using observed regional yields to describe stochastics and to design different insurance variants. The optimal uptake rate of the insurance is found to depend on its attributes (strike level, costs) and to interact with adjustments in the farm program. The latter impact of crop insurance was so far not considered in studies which analyzed new or existing insurance products under CPT, assuming fixed farm management.

From a numerical perspective, the implementation requires integer variables which provokes a sizeable increase in the computational burden. For optimization model so far not using integers, a change in the required solver is likely. An implementation in models with non-linear constraints which are not quadratic might prove challenging as the set of available general mixed non-linear solvers is still limited.

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